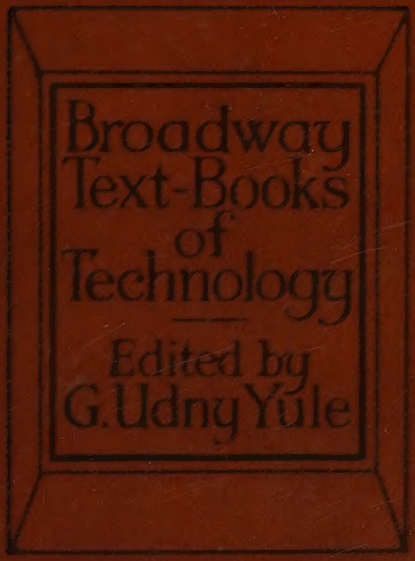
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WITH AN INTRODUCTION BY

C. H. REILLY, M.A., F.R.I.B.A.

PROFESSOR OF ARCHITECTURE AT THE UNIVERSITY OF LIVERPOOL

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INTRODUCTION

By C. H. REILLY, M.A., F.R.I.B.A., *Professor of Architecture in the University of Liverpool*

THE Board of Education in recent years has permitted teachers in building construction a far greater latitude in their methods of work than was formerly the case. Where there have existed teachers with practical knowledge of building, and the power to impart that knowledge, the results have been highly satisfactory; a new vigour has been infused into the work, and theory has once more been related to, and tested by, actual experience. This is what has happened in the branch technical schools in Liverpool. A group of teachers, combining enthusiasm with practical knowledge of building conditions, have gradually evolved new courses of study and demonstrated the success of their methods. These teachers have now been persuaded to bring such portion of their teaching as can be reduced to text-book form within such limits, and this series of handbooks is the result.

Briefly, the scope of the authors' design has been to provide text-books of geometry, building construction, and building materials respectively, and so to relate them to one another and to practical work that the student shall feel from the outset the reality of the study he has undertaken. For instance, the simplest as well as the more advanced examples in descriptive geometry are chosen from actual building problems. The boy who has not the power to grasp abstract form is taught to realise the same form through the familiar shapes of buildings and their parts. In the same way care has been taken that the examples of construction given are from ordinary simple buildings. The imaginary

bay windows and impossible dormers of the text-book will not be found in them. While the examples given show all the necessary construction in the fullest possible way, the authors have wisely restricted themselves to the simplest forms, and where it has been impossible to avoid architectural expression altogether, they have chosen such forms as show the warrant of a wide tradition rather than individual idiosyncrasies of modern design.

Everywhere in the British Isles, even in the latest manufacturing districts, there are at hand numberless examples of simple eighteenth and early nineteenth century buildings. Such cottages, farmhouses, and even shops abound. In these the building construction student will find the sash window, the two- the four- or even the six-panel door, and all the other elements of modern building in their pristine simplicity. They are there in the form which tradition and use have brought about before the builders of the modern suburban villa had attempted the impossible task of giving such things individual expression. If the teachers of building construction throughout the country would but direct their students to such examples, the modern suburbs of our big towns would soon cease to depress us. These text-books seem to me a good step in that direction.

AUTHOR'S PREFACE

It is presumed that the student who commences the study of this volume has already carefully perused and systematically worked through the First Year Volume of the same series, or has satisfactorily passed through some approved first year course of instruction.

Many of the fundamental principles contained in vol. i. will be constantly considered in relation to further and special applications to builders' work, and in many cases the examples will follow in logical sequence introducing further practical difficulties. Hence students are advised to have vol. i. constantly at hand when following private study, so that speedy reference may be made as occasion demands.

Building and architectural students should try to realise from the outset that the value of geometry as a groundwork for nearly all their future study cannot be over estimated.

It supplies their means of representing mental conceptions and communicating these to others, and provides a basis for the solution of constructive problems of all kinds.

A thorough knowledge of geometry and practical mathematics opens the only way to the satisfactory study of advanced construction and design.

Although the intention of this work is to concentrate on practical geometry as applied to building, examples of calculation in relation to area and volume will be inserted as occasion arises.

In following the course, students are advised, *where necessary*, to construct paper or cardboard models of parts of the structure with which they are dealing, in order to obtain a sound grip

of "form" and "position" relating to lines, planes, and solids, but they are also warned that model-making is only a means to an end, and that end is to engender and develop the power to visualise the relation of lines and planes to each other, and to recognise them in structural work as well as on paper.

Having overcome fundamental difficulties in connection with the representation of planes, measurement of inclination, etc., the student is then advised to abandon models—except as a last resource—and to depend on mental grasp of the conditions, helped out possibly by freehand sketches in isometric or pictorial projection, such as occur from page to page in this book.

He must realise that he has only become master of his subject when all such external aids to orthogonal representation may be put aside and the problem visualised, gripped, and solved at once by the use of ordinary projection.

Many examples are given in the following chapters, sufficient to afford the teacher a suitable choice for class work and home work, and yet leave the student enough exercises for private study, which it is our intention to encourage in every possible way.

F. E. DRURY.

Royal Technical Institute,
Salford, *December* 1914.

ABBREVIATIONS AND NOTATION USED IN THIS VOLUME

(*In general agreement with vol i.*)

H.P. = Horizontal plane, or ground plane.
V.P. = Vertical plane.
S.V.P. = Secondary vertical plane (at 90° to H.P. and V.P.).
A.V.P. = Auxiliary vertical plane.
A.H.P. = Auxiliary horizontal plane.
θ (theta) = Slope of a line or plane to the ground or H.P.
ϕ (phi) = Slope of a line or plane to the vertical plane.
β (beta) = Slope of a line to a "vertical line."
H.T. = Horizontal trace.
V.T. = Vertical trace.

Capital Letters generally represent actual points in space.

Small Letters represent the plans of such points.

Small Letters with an *index*, as a^1, a^2, a^4, etc., represent elevations. All points indexed a^1 are contained in the *primary* or principal elevation; points indexed a^2, belong to a second elevation, and so on, according to the number of elevations used.

Small Letters with a suffix, as a_1, a_2, etc., refer in a similar way to plans which may be auxiliary to the principal plan.

Capital Letters with suffix $_0$, as A_0, B_0, etc., represent points which, in the solution of a problem, have been rotated or rebatted into the H.P.

Capital Letters with suffix $_v$, as A_v, B_v, represent points rebatted into the V.P.

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SECOND YEAR GEOMETRY OF BUILDING CONSTRUCTION

CHAPTER I

AUXILIARY PROJECTION—ELEVATIONS

We shall assume that you know how to represent any simple solid or structure in orthogonal—or orthographic—projection, viz., by plan, elevation, and vertical plane section, and that you have grasped the fact that the object of such projection is to supply all "necessary" information regarding the shape, size, and position of every part of such object. Further, that practical work demands an adequate representation *only*, not *over* representation, which might possibly supply more, but unnecessary information.

It is quite unscientific to waste time over the solution of a problem in practice, though you must recognise for the present that we shall very often need to reach our "final method" by processes, part of which at least may be discarded, when we are really conversant with our work.

Observe further, that geometrical notation is only necessary for explanation to the learner, and that this may, and should be, abandoned, when persons obtaining and perusing the result do not require such assistance.

Projection

In your earliest consideration of projection you found that "a plan shows the true lengths of horizontal lines only," viz., lines contained in the horizontal plane or parallel thereto, and that "an elevation shows the true lengths of vertical lines and those contained in, or which are parallel to, the vertical plane." Think carefully over the latter statement, and make sure of the positions of the lines referred to.

Now many objects and structures are of such a form that

they contain lines—and surfaces—which, in no ordinary projections show their true length—or true shape. By this we mean that the common position in which they are assumed, and the method of representing them, leaves some edge or face of the solid imperfectly represented.

The constructor, and often the designer, needs to know the real length of a line in a structure, or the true shape of a surface, in order to prepare material for assembling in its erection. We have, therefore, to devise processes which will enable us to represent objects so that the true forms of such parts may be seen in one of our "projections," or, at a later stage, we may probably reconstruct the outline of that portion imperfectly represented, so as to obtain the same result.

There are two general principles employed in solving such problems, viz.:—

(*a*) Auxiliary projection.
(*b*) Rotation, or rabatment of surfaces into convenient planes for truthful representation.

We shall concern ourselves at present with the former method—auxiliary projection.

Definition. — An auxiliary projection is a view — or projection—of any line, plane surface, or solid *on a plane other than the ordinary planes* of projection.

For our purpose we shall include in the "ordinary" planes, the horizontal plane (H.P.), the vertical plane (V.P.), and the secondary vertical plane (S.V.P.), on which "side elevations" have been projected.

Any plane "other than" these three, we shall call an "auxiliary plane."

If the plane be vertical we shall further define it as an auxiliary "vertical" plane, and refer to it as A.V.P.

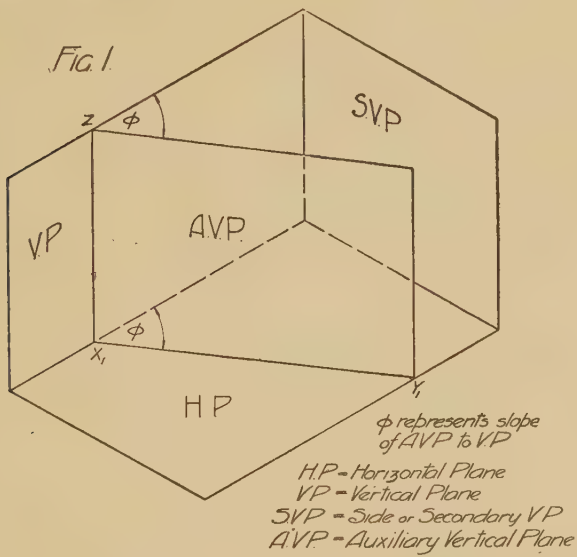
Let us confine our attention in the rest of this chapter to elevations drawn on such planes; then, as all elevations are projections on vertical planes, the whole of our views may be termed "auxiliary elevations."

The first principles of auxiliary projection have been referred to in the first year volume at pp. 103 and 104, where new views were obtained on chosen ground lines.

Having studied these you will know the process by which such views are constructed, but we must now go further and show "why" such procedure is followed, and then apply it to more difficult problems which allow of its use.

Examine Fig 1. This represents the H.P., V.P., and S.V.P. in position, with a fourth plane (A.V.P.) added to them, being perpendicular to the H.P. but inclined at the angle ϕ to the

V.P. It cuts the H.P. and V.P. in the lines—or traces—X_1Y_1 and X_1Z respectively, both of which might be continued on opposite sides of the two planes should the problem call for it. We have named X_1Y_1 the ground line of the auxiliary plane. Later it may be more conveniently referred to as the horizontal trace (H.T.).



Any object projected on to this auxiliary plane produces an *elevation*, because the plane is vertical.

Now refer to Fig. 2, which is an isometric view of a quoin stone for the obtuse angle of a building, resting on the ground plane with the plan outlined underneath the block and an elevation projected on the V.P. These two views are orthogonally represented in the plan and front elevation of Fig. 3, and from them we gather the exact shape of the top and bottom surfaces and the vertical height of the block, and find that the top and bottom surfaces are parallel; but the vertical surfaces do not show their true shape, except in the case of the back surface.

Suppose we require to view the quoin so as to show the true shape of the face marked A, it is clear that we must view the block at right angles to the surface A, or, to be more explicit, *we must make a projection on a plane "parallel" to A.*

The direction of view for this projection is shown by the arrow on Figs. 2 and 3, and the auxiliary plane on which the projection is to be made is at 90° *to the line of view.* The process of projection is seen to be exactly the same as we

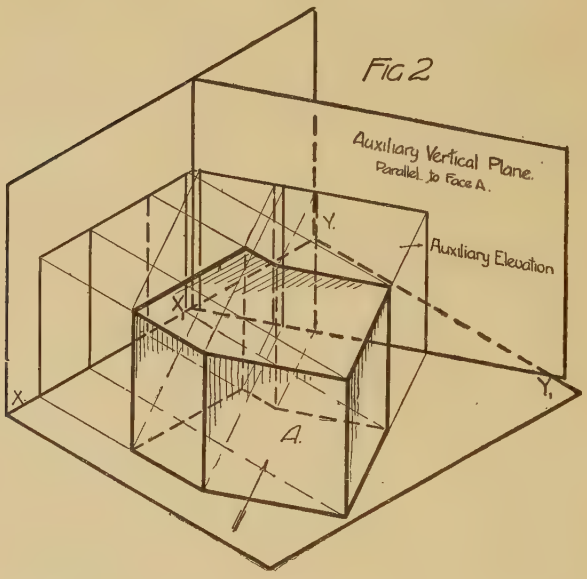
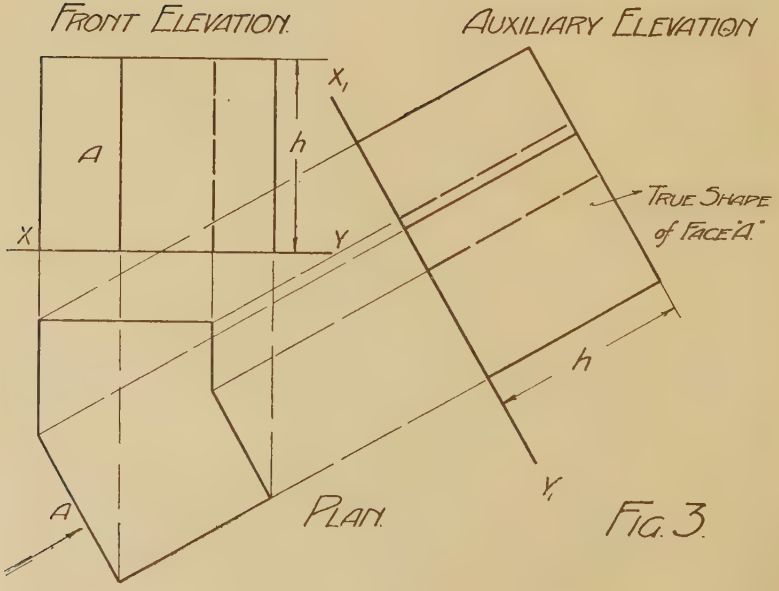


FIG 2
Auxiliary Vertical Plane.
Parallel to Face A.
Y.
Auxiliary Elevation
X.
X,
Y,
A.


FRONT ELEVATION.
AUXILIARY ELEVATION
X,
A
h
X
Y
TRUE SHAPE
of FACE 'A'
h
A
PLAN.
Y,
FIG. 3.

employ in obtaining *any* elevation, viz., the prominent points are projected at 90° to the plane—their heights thus remaining the same as in the primary elevation—and the angles joined to portray the solid. If the auxiliary plane be now rotated about its ground line, X_1Y_1, until it rests on the H.P. with the drawing upon it, its appearance and position is that of Fig. 3.

Observe here that *so long as the position of the solid is the same, all elevations of it must show its prominent points of the same height*, and, as the elevation must lie opposite to, or upon the projector from the plan, we get the following simple rule :—

To construct an "auxiliary elevation" project from the plan at 90° *to the new XY and measure the heights of the projected points beyond the new XY from the corresponding points in elevation.*

This is often condensed to :—

"PROJECT" FROM PLAN AND "MEASURE" FROM ELEVATION.

Take careful thought regarding the relationship of these views. To grasp their relationship, the auxiliary elevation and

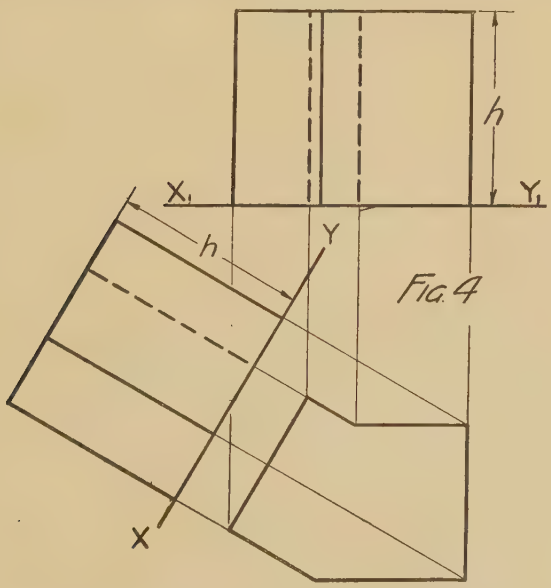


FIG. 4

its plan must be viewed at 90° to X_1Y_1 and the primary elevation disregarded. These are redrawn at Fig. 4, with the paper turned so that X_1Y_1 is in a horizontal position, showing that

the plan and auxiliary elevation bear the same relation to each other as any other pair of projections.

As we proceed with our course of study you will see that "auxiliary elevations" are conveniently used for the following purposes :—

(*a*) To show the true shape of any figure or surface which lies in a vertical plane.
(*b*) To present the true length of any "inclined line" contained in a vertical plane.
(*c*) To present an "edge view of a plane."
(*d*) To obtain the true shape of a section made by a vertical plane (*see* vol. i., p. 106).
(*e*) To enable us to visualise the appearance of any detail of a structure as viewed in a given horizontal direction. (This often affects the design of irregular masses on a level with the eye.)

Having had some practice in your first year course entailing the process of drawing auxiliary elevations, you should now work through the following examples, noting in connection with each the particular object in view.

We shall deal in later chapters with "sections" and "edge views of planes."

EXAMPLES ON AUXILIARY ELEVATION

(1) Fig. 5 gives the plan and elevation of a pyramidal roof, 18 ft. square at the base and 34 ft. high. Project an elevation of the roof which shall show the true length of the angle rafter or "hip" (AV), the plan of which is *av*. Measure and state the true length. Check this by calculation, using the property of the squares on the sides of a right angled triangle.

Answer, *av* = 12.72 ft.
AV = 36.3 ft.

(2) In Fig. 6 three views of a terra-cotta coping block are supplied. This occurs at the junction of two walls at an angle of 135° in plan. Reproduce these views to a good scale, and add a third projection in the direction of the arrow (which is at 90° to the mitre lines where two sloping surfaces meet). Find, from your projection, the length of the line of intersection.

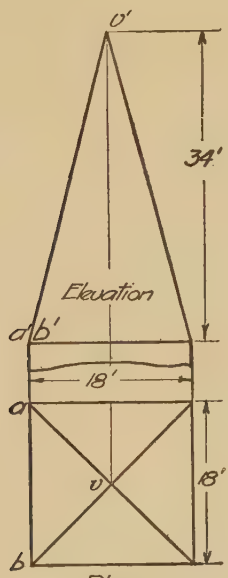

0'
34'
Elevation
a'b'
18'
a
v
18'
b
Plan

Fig. 5.

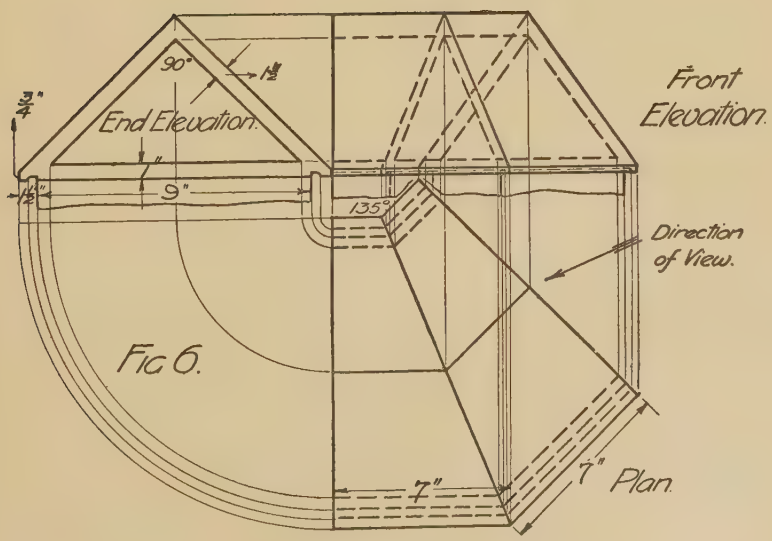
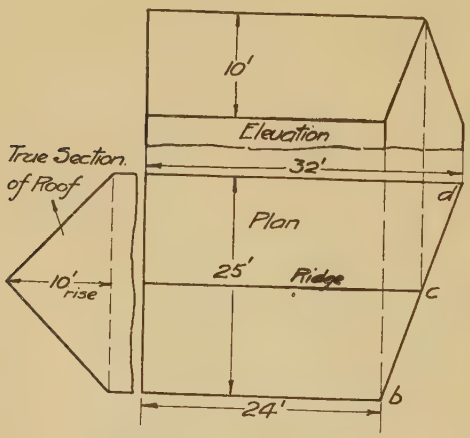

90°
1½
¾"
End Elevation.
½"
9"
135°
Front
Elevation.
Direction
of View.
7"
7" Plan.

Fig. 6.



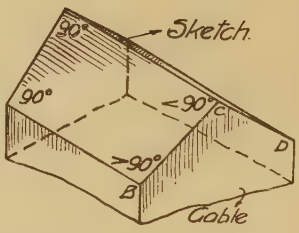


Fig. 7.

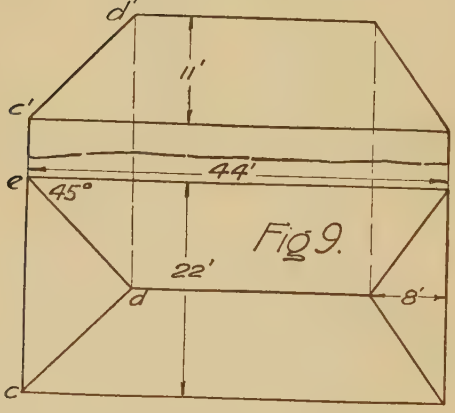


Fig 9.

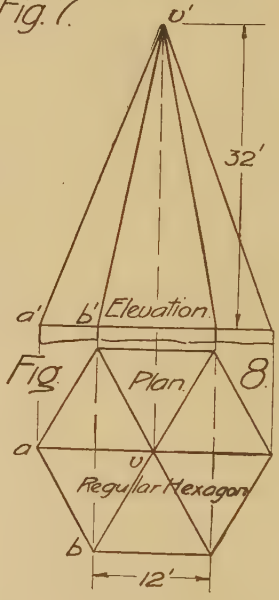


Fig. 8.

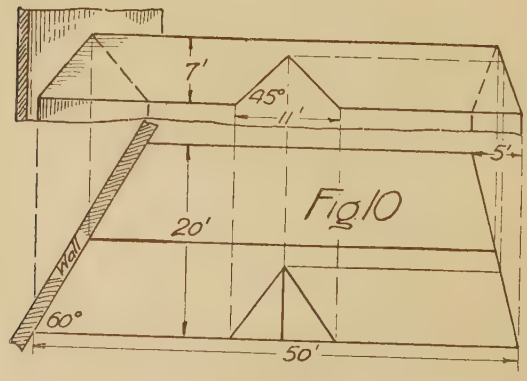


Fig10

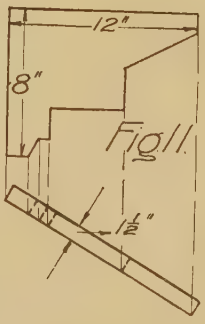


Fig11.

(3) Three projections of a roof to a building, one end of which is square and the other splayed, are given in Fig. 7. A sketch is also added. From the information supplied project an elevation of the whole roof so as to include in the projection a "full view" of the splayed gable. What information does this supply which is not available in the given drawings?

(4) Fig. 8 gives the plan and elevation of a hexagonal spire roof. By an auxiliary elevation obtain the edge view of the surface ABV.

(5) Plan of a hipped roof, Fig. 9. The hipped ends are of different slopes, hence their plans are of different shape. Obtain an elevation on a V.P. so that the hip CD appears as a vertical line. Is the length of DE as seen in your new elevation the "true length"? Give your reasoning.

(6) Fig. 10 is an irregular roof with a splayed gable at one end; it abuts on a splayed wall at the other. In the centre is a small "cross gable." Project a view of this roof in a direction at 90° to the wall at the left hand.

(7) Fig. 11 gives a plan and the elevation of the "front face" of a rough bracket for a large cornice. Complete the elevation to half scale, given that the edges are cut square to the face. Construct an elevation from these views to show the true shape of the bracket.

(8) The true cross section of a cornice moulding is represented in Fig. 12. Required, the form at a 45° mitre as shown by the single line in plan.

NOTE.—This problem is identical with one given in a later chapter on enlarging mouldings. In this instance, treat it merely as an auxiliary elevation at 90° to the plan line. To obtain the "curves" take intermediate points, project and measure the position of these like the angular points, and connect by fair curves similar to those of the original figure.

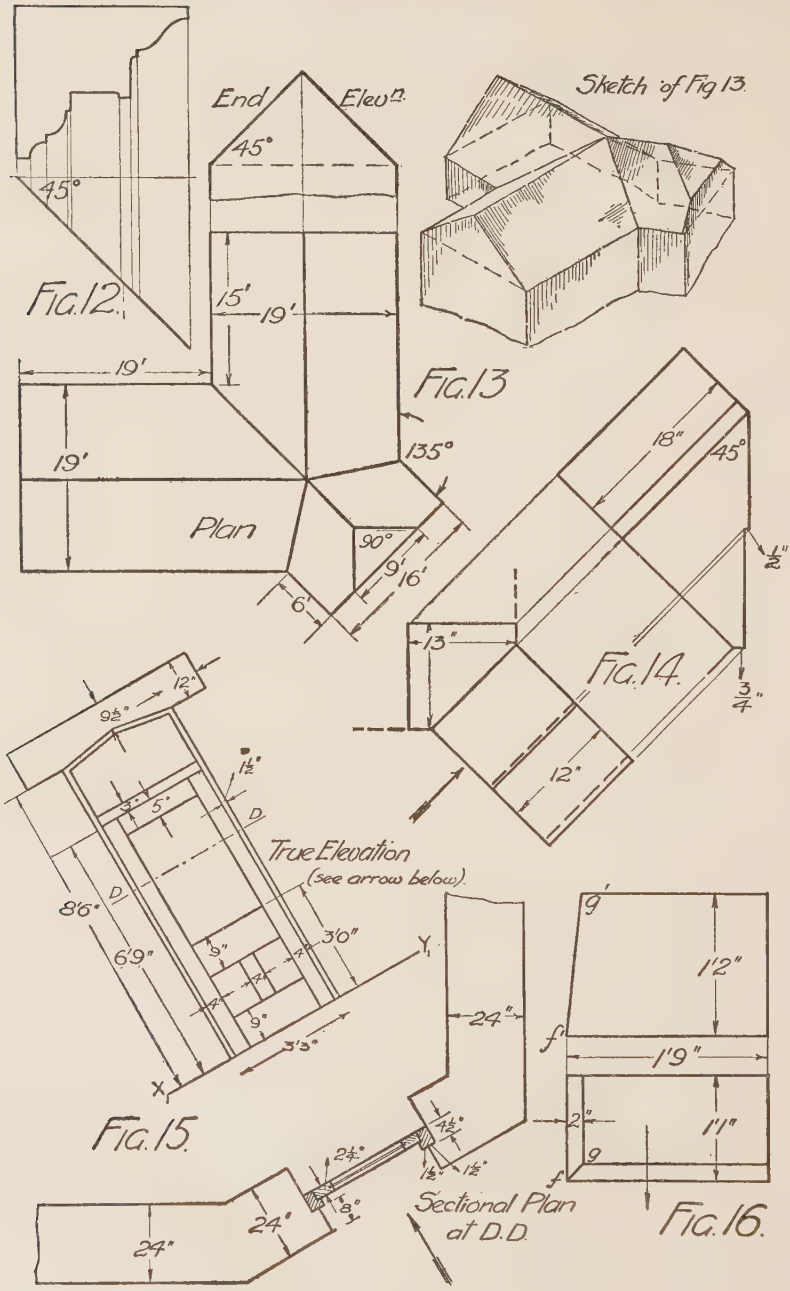
(9) Plan and cross section of an L roof with a wing projecting at the external angle (Fig. 13). Project a view at 90° to the angle gable. To grasp the form properly, refer to the sketch on the right, and observe also that the ridges are all of the same height, which may be determined from the end elevation.

(10) In Fig. 14 is illustrated the plan and side elevation (*see* arrow) of the top stone for an angle buttress. Draw a true elevation of the stone viewed at 90° to the wall face, which is shown by the dotted horizontal line.

(11) A doorway is formed at the angle of a building as per Fig. 15. The right angle is cut away 7 ft. on the front and 4 ft. along the side wall. A horizontal section of the wall and doorway is given, and a true elevation of the latter showing the opening, lintel, frame, and door. All mouldings are omitted. Draw the elevation as it would appear looking directly towards the front wall.

NOTE.—*In problems* 10 *and* 11 *you are utilising what appears to be an auxiliary elevation to obtain a primary elevation. It is wise to think of the given elevation as the primary one, merely placed in a convenient position for the projection of the required view in its most natural position.*

(12) Fig. 16. A quoin stone in a wall with a sloping, or "battered," face, such as occurs in a retaining wall or at the angle of a spire. Draw the elevation viewed at 90° to the angle FG.


End Elevn.
45°
45°
15'
19'
19'
19'
Plan
135°
90°
9'
16'
6'
Fig. 12.
Sketch of Fig 13.
Fig. 13
18"
45°
½"
13"
¾"
12"
Fig. 14.
12"
9½"
1½"
3"
5"
D
D
8'6"
6'9"
True Elevation
(see arrow below).
9"
4"
4"
9"
3'0"
3'3"
Y
X
Fig. 15.
24"
1½"
2½"
2¼"
1½"
8"
24"
24"
Sectional Plan
at D.D.
9'
1'2"
f
1'9"
2"
1'1"
9
f
Fig. 16.

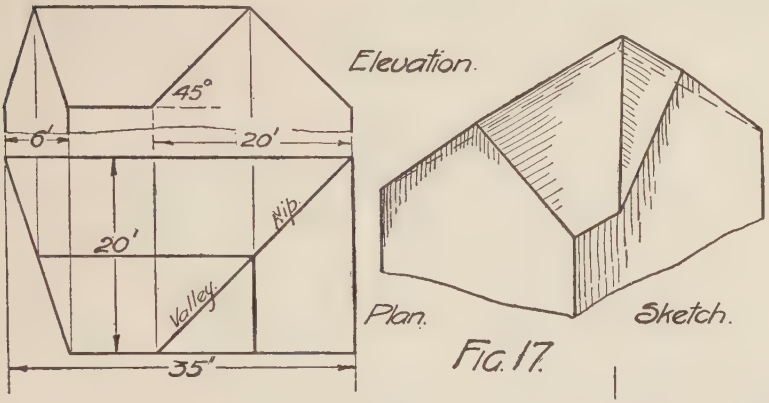


Fig. 17.

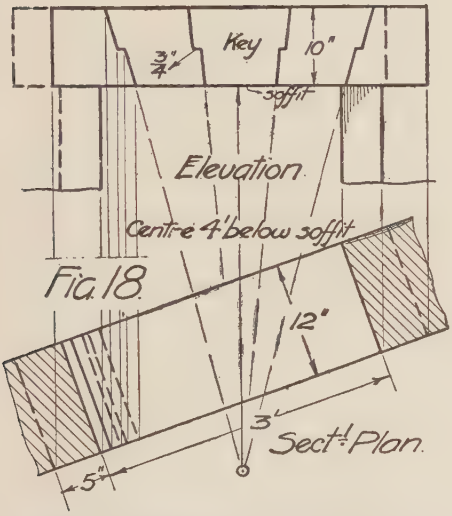


Fig. 18.

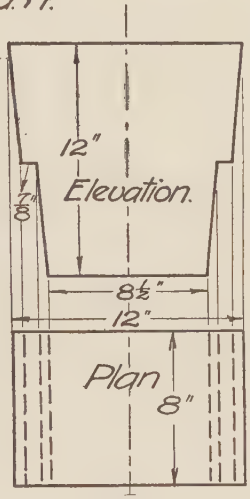


Fig. 19

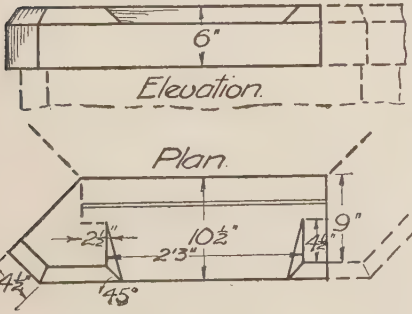
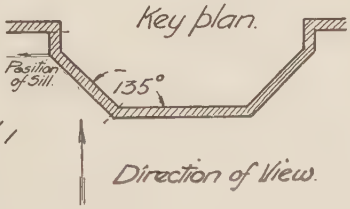


Fig. 20.



(13) Projections and sketch of a gabled "hip and valley" roof with a splayed end (Fig. 17). Obtain an elevation viewed horizontally in the direction of the valley. Does this view serve any useful purpose? If not, can you suggest a view that would incidentally do so?

(14) From Fig. 18 determine the true elevation of the flat window arch. All joints to be clearly shown.

(15) Fig. 19 is the centre or "key" stone of an arch such as we have considered in problem 14 (Fig. 18). If this key is contained in an arch, whose face slopes at 30° to the direction of view, draw its elevation.

(16) In Fig. 20 is illustrated a window sill for the splay window of an octagonal bay. Its position may be understood by reference to the key plan on the right. Draw an elevation of this as it would appear in the front elevation of the window.

CHAPTER II

AUXILIARY PROJECTION—PLANS

AN "auxiliary plan" bears the same relation to an ordinary plan that an "auxiliary elevation" bears to a primary elevation

We defined an auxiliary elevation as a projection on any vertical plane *other than* the ordinary V.P. We cannot, however, define an auxiliary plan in quite the same fashion, because we can only have "one" horizontal plane for purposes of projection. But, observe that the horizontal plane is at 90° to the vertical plane, and we could turn the plane so that it is no longer horizontal while still maintaining its right angle to the V.P. A view on such a plane (inclined to the horizontal, but at 90° to the vertical) is called an auxiliary plan, and is obtained by "viewing the object in a direction parallel to the V.P., but at 90° to the auxiliary plane"—*not* in a vertical direction.

The auxiliary plan may therefore be defined as "a projection on any plane which is at 90° to the V.P. *other than* the H.P."

Thus the "auxiliary horizontal plane" is really the simplest kind of inclined plane whose inclination to the horizontal is denoted by θ (theta). Fig. 21 is a pictorial or conventional illustration of an "auxiliary H.P."

The lines of intersection of the A.H.P. with the V.P. and H.P. are referred to as traces, and the ground line—X_1Y_1—of the A.P. used to represent the plane for auxiliary projection of plans is really the "vertical trace."

Fig. 21*a* illustrates the usual position of the auxiliary plane *below* the H.P., which is commonly adopted for the projection of auxiliary plans.

Suppose any object to be placed in the angle between V.P. and H.P., and an auxiliary plane be inserted as in Fig. 22. Let a pair of projections be made on V.P. and H.P.; then, without moving the object, a third projection be constructed on the A.H P. whose ground line is X_1Y_1. Now let the A.H.P., with the projection upon it, be turned about its ground line (X_1Y_1) until it lies in the V.P. Lay out the V.P. and H.P. as usual, and you have the three views represented in Fig. 23.

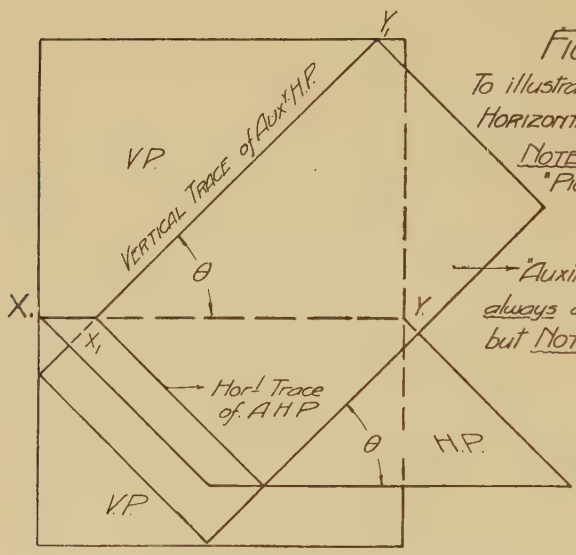


FIG. 21.

To illustrate "AUXILIARY HORIZONTAL PLANE," (A.H.P.)

NOTE:- This View is a "PICTORIAL PROJECTION."

→ "Auxiliary H.P." – is always at 90° to V.P. but NOT Horizontal.

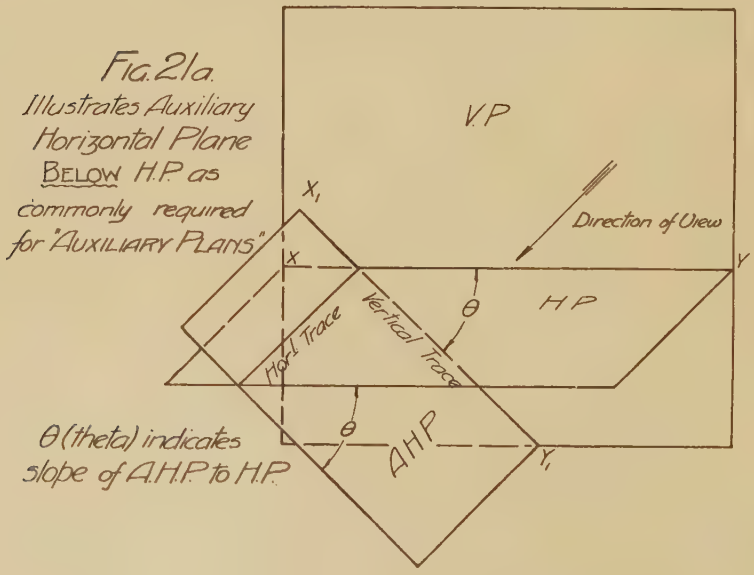


FIG. 21a.

Illustrates Auxiliary Horizontal Plane BELOW H.P. as commonly required for "AUXILIARY PLANS."

θ (theta) indicates slope of A.H.P. to H.P.

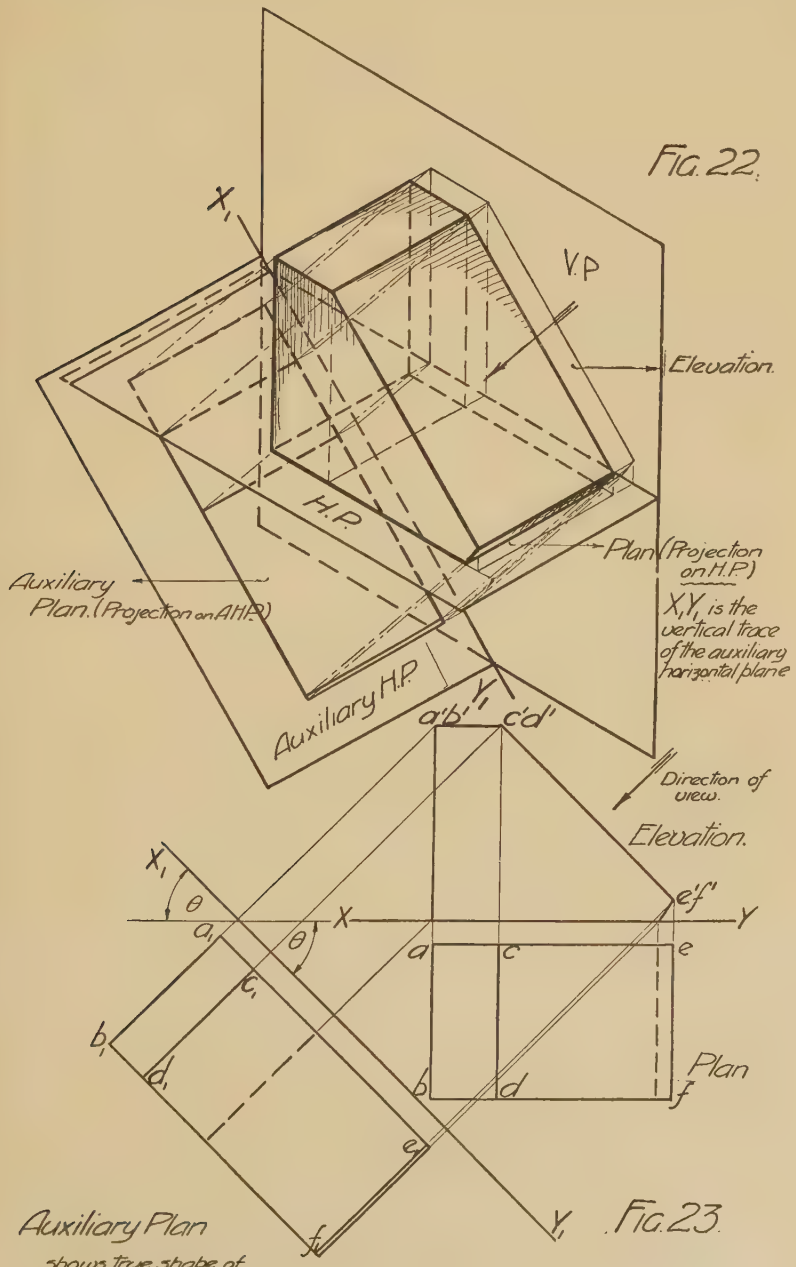


Fig. 22.
X1
V.P
Elevation.
H.P.
Plan (Projection on H.P.)
Auxiliary Plan. (Projection on A.H.P.)
X1Y1 is the vertical trace of the auxiliary horizontal plane
Auxiliary H.P
a'b'
Y1
c'd'
Direction of view.
Elevation.
X1
θ
a1
X
Y
e'f'
θ
a
c
e
c1
b1
d1
b
d
f
Plan
e
f1
Y1
Auxiliary Plan
shows true shape of surface C.D.E.F.
Fig. 23.

Study Figs. 22 and 23 together, and make certain that you have grasped the operation Notice that X_1Y_1—showing the slope of the A.H.P. to the horizontal—is correctly seen in position by its edge view in the elevation only; also, that the projectors for the auxiliary plan have their true lengths seen in the elevation. But their definite position can only be obtained by reference to the plan, for this shows the distance of each point in front of V.P.

Now try to realise, that when the three planes are in actual position, the X_1Y_1 is the edge view of the whole plane A.H.P., that projectors from the elevation to this plane would stop at X_1Y_1, but, when rotated about this line so that the plane lies flat in the V.P. the circular path of each point would be seen as a straight line in "edge view," and *each would move as far BEYOND the X_1Y_1 as it stands in FRONT of the V.P.*

From this consideration we arrive at the following conclusion:—

> *To construct an "auxiliary plan" we must place the X_1Y_1 of the auxiliary plane in a position at 90° to the direction of view desired, project each elevation point at 90° to this X_1Y_1 and beyond, and on each projector measure the distance of the point in front of X_1Y_1 to correspond with the original "plan distance" in front of the V.P.*

The rule for carrying out this process is often abbreviated to:—An auxiliary plan is obtained by "PROJECTING" FROM THE "ELEVATION" AND "MEASURING" FROM THE "PLAN."

Having obtained the positions of the angles of the solid or other desired points, you must mentally picture the "form" of the object as viewed in the desired direction and join up the angles to produce the outline, noting invisible edges and representing these, as usual, by dotted lines.

Auxiliary views may be produced by observing that the *same angles* are joined by the *same kind of line* in every view of the object. Thus, if pairs of corresponding points in two views are obtained there is no difficulty in placing the line between them; on the other hand, only mental conception can determine whether the lines must be full or dotted, and what the external outline of the new projection will become.

Any such work as this is very imperfectly done if mechanical determination and connection of points is the nett result of the student's efforts. The object is to develop capacity to visualise form and position of solids, etc., with whose "general shape" the student is first made familiar.

The use of models may be recommended in the first stages for explaining the position of the solid which is to be projected, but to rely too long on this assistance is to retard development. It is wiser to *attempt* the projection and *check results* by the use of models.

NOTE. — Auxiliary plans are much more difficult to conceive than auxiliary elevations, and most careful attention and concentration on the principle is desirable in order to recognise its possibilities for practical application.

Figs. 22 and 23 illustrate one of the prime uses of an auxiliary plan, viz., to obtain the true shape of "inclined surfaces," which, because of their inclination, do not appear the true shape in plan. Their edges must appear as *lines* in the elevation.

In Fig. 23 the true shape of the top-sloping face of the block is exhibited, because the auxiliary plan is constructed on a plane parallel to that face. Note that the "edge" of the face only was visible in the elevation. This is an important condition.

USES OF AUXILIARY PLANS

Some of these uses have been described under the heading of auxiliary elevations. That is to say, we may have the same object in view, but the position of the part to be represented in true shape determines whether a *plan* or an *elevation* is required.

The following summary will indicate the chief uses :—

(*a*) To obtain the true shape of sloping surfaces whose edge views are seen in elevation. (In this case we employ an A.H.P. parallel to the surface whose true shape is to be determined.)

(*b*) To obtain the true shape of a section made by an inclined plane (*see* chapter on "Sections").

(*c*) To obtain the true lengths and inclinations of oblique lines (*see* later chapters).

Although the auxiliary plan is more limited in its use than the auxiliary elevation, it is of inestimable value in solving constructive problems and in developing the power to visualise the appearance of objects in different positions.

You should now proceed to the following problems involving the use of the "auxiliary plan."

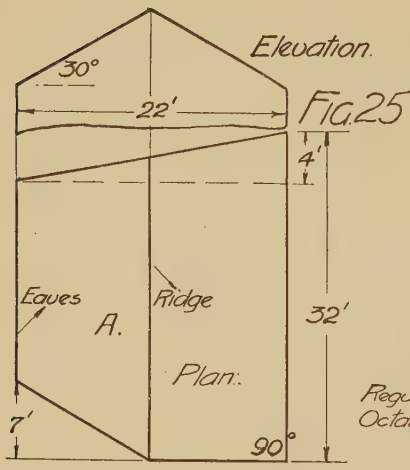


Fig. 25

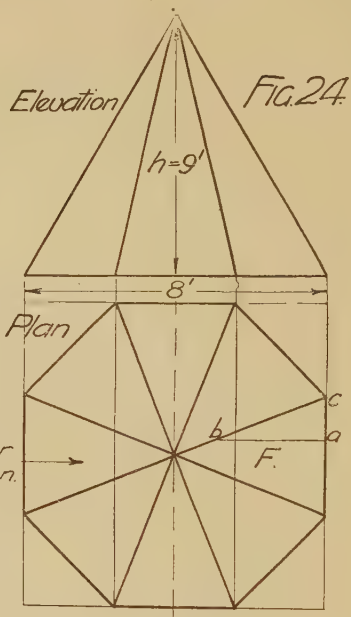
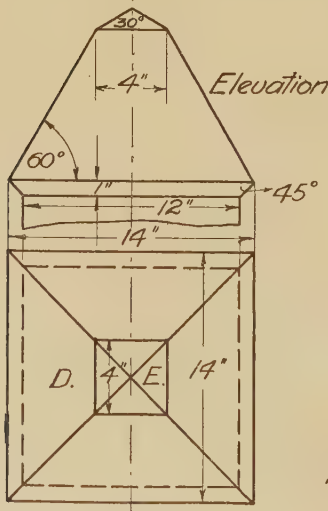


Fig. 24.



Plan

Fig. 27.

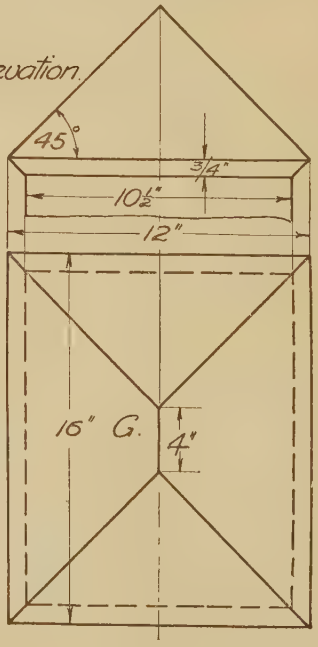


Fig. 26.

Plan.

EXAMPLES

(Combining Auxiliary Plans and Elevations)

(1) Reproduce the roof in Fig. 25. From the given views project an auxiliary plan of the surface A so as to determine its true shape. The plan is to be complete.

(2) Fig. 24 represents a regular octagonal pyramidal roof. Required, an auxiliary plan of the whole roof, to include the true shape of the surface F, and the true length of the line AB marked upon it.

NOTE.—The view will also show the angle between AB and BC, if AB is reproduced in the new plan.

(3) Fig. 26 represents a stone cap for a rectangular gate pillar. Project a complete plan of the cap showing the face G in true shape.

(4) A stone gate-pier cap is shown in Fig. 27. Draw the auxiliary plan that will show the true shape of the face D, and a further plan to exhibit the true shape of face E. Construct an auxiliary elevation of the cap viewed in a direction making 60° with the front edge in plan. Notice that the form of this cap is a combination of pyramids or parts of them.

(5) Project a new plan of the piece of connecting steel angle in Fig. 28. The direction of the required projection is indicated.

(6) Fig. 29 shows three views of an angle plate, such as may be used for connecting a light stanchion to its bed-plate. Obtain an auxiliary plan of the angle viewed in the direction of the arrow.

NOTE.—*Views of the nature of those required in problems* 5 *and* 6, *occur in drawings of the connections of inclined members in steel roofs, etc.*

(7) Cast-iron shoe for the foot of a wooden principal rafter, Fig. 30. It is prepared to receive a $1\frac{1}{8}$-in. tie bolt. Removing the timber rafter, draw a new elevation viewed horizontally at 45° to the face of the casting.

(8) The connection at the foot of a steel T used as a roof strut is shown in Fig. 31. Draw a new plan at 90° to the table of the T (*see* arrow). Add a $\frac{7}{8}$-in. diameter hole at the centre of the "foot."

(9) Fig. 32 is the plan of a triangular hole in a surface inclined upwards from left to right at 45°. By auxiliary projection obtain the true shape of the hole. If the slope be altered from right to left upwards, what alteration takes place in the shape of the hole?

(10) Plan and side elevation of a weathered stone cap to an angle buttress, Fig. 33. Required, a view to show the true shape of the surface H. Make a freehand sketch of this stone and also an elevation square to the main wall.

(11) Fig. 34 is the hipped end of a lean-to roof. Prepare a drawing of the surface H only, to indicate its true shape.

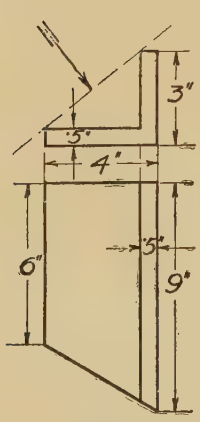


Fig. 28.

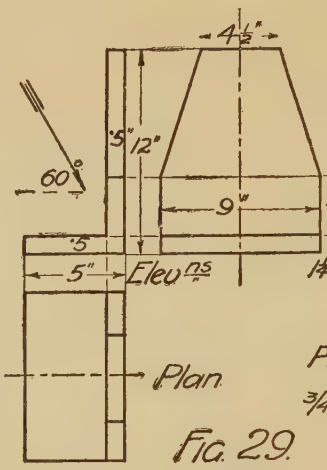


Fig. 29.

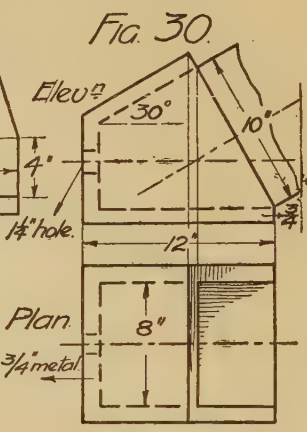


Fig. 30.

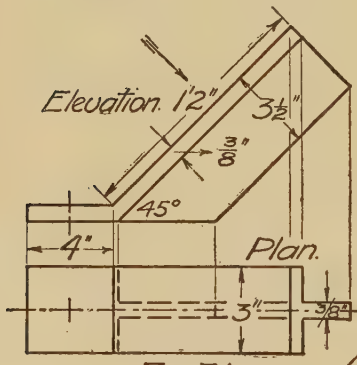


Fig. 31.

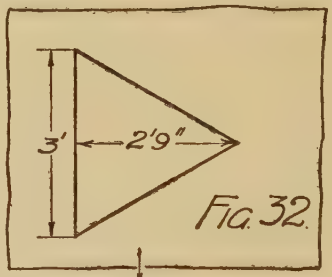


Fig. 32.

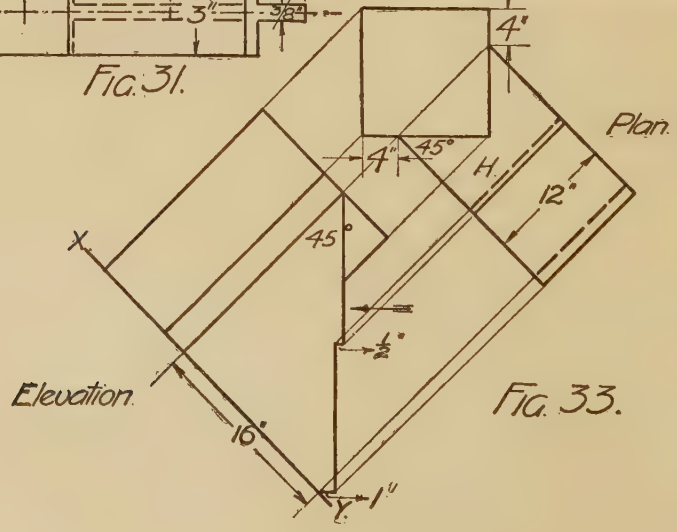


Fig. 33.

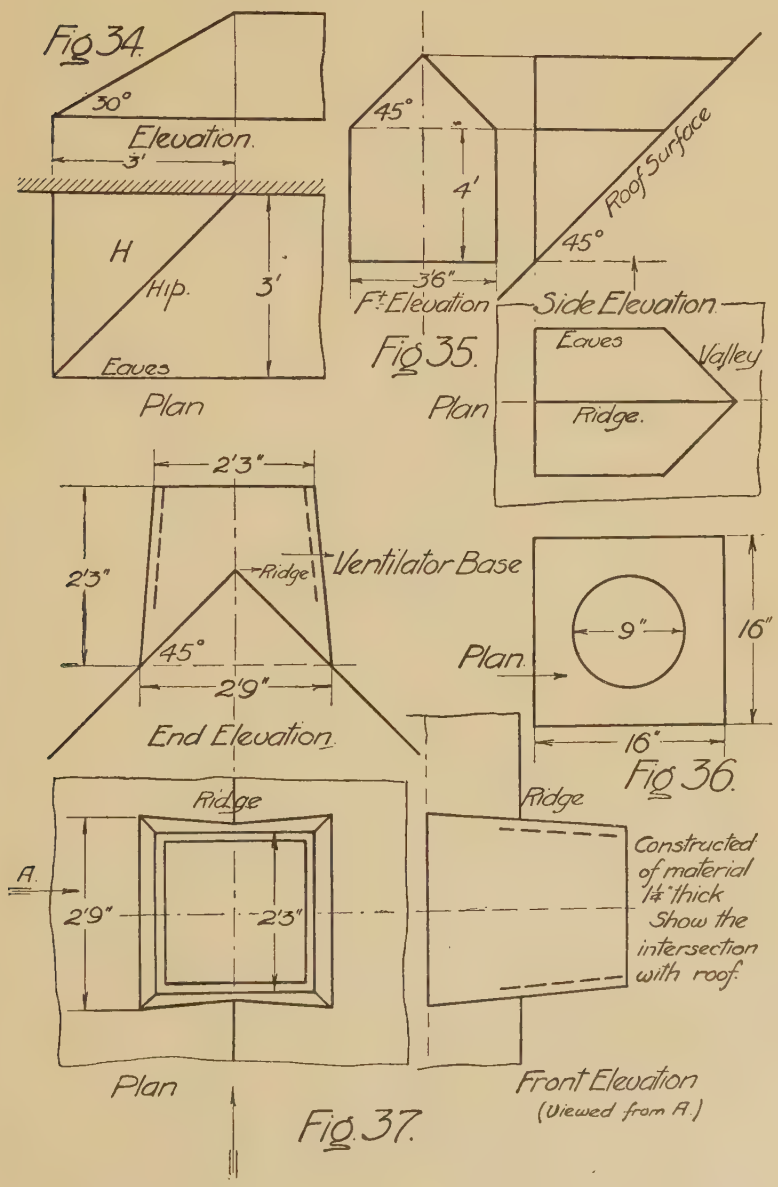


Fig 34.
30°
Elevation.
3'
H
Hip.
3'
Eaves
Plan
45°
4'
36"
Ft Elevation
Roof Surface
45°
Side Elevation
Fig 35.
Eaves
Valley
Plan
Ridge.
2'3"
2'3"
Ridge
Ventilator Base
45°
2'9"
End Elevation
Plan
9"
16"
16"
Fig 36.
Ridge
Ridge
A.
2'9"
2'3"
Constructed of material 1/4" thick Show the intersection with roof.
Plan
Front Elevation
(Viewed from A.)
Fig 37.

(12) The three views of Fig. 35 represent in single lines the shape of a "dormer," erected on a roof inclined at 45°. By any arrangement of drawings you please, based on auxiliary projection, determine the true shape of one of the roof surfaces.

(13) Fig. 36 is the plan of a sheet metal plate through which a vertical pipe passes. The plate is fixed in a roof whose inclination is 30°. Find the true shape of plate and hole by an auxiliary plan.

NOTE.—*Take several points in the plan of the circle, symmetrically arranged, project these and draw a fair curve through them.*

(14) Three views of a ventilator base on a vee roof, Fig. 37. Complete these views from the particulars given, taking into account the thickness of the material. Project the true shape of each sloping face (without repetition).

CHAPTER III

SECTIONS (BASED ON AUXILIARY PROJECTION)

THE construction of "sections" and "sectional elevations" has been discussed at pp. 107 and 108 in vol. i.

It was there explained that sections are employed to exhibit the internal construction of objects, and that "a section is the surface formed by the intersection of a plane and a solid."

In the first volume the explanation was confined to the construction of sections made by vertical planes, and of sectional elevations where visible parts of the solid *beyond* the cut surface were shown, assuming the forward portion to be removed.

We must now proceed to obtain a further knowledge of the principles on which sections are constructed.

Observe first that sections and sectional elevations previously considered were, in the nature of their construction, auxiliary elevations of the sectional surfaces, or of these and some additional parts.

But the cutting plane *may not be a vertical plane.* It may be an "inclined" or even an "oblique" plane.

As an illustration consider Fig. 38 which supplies three views of an octagonal prism. This prism seldom occurs horizontally in building structures, but will serve our purpose very well as an illustration. It is cut by a plane whose "edge view" is EF, viz., an inclined plane at 90° to the V.P. EF is equivalent to the ground line of the cutting plane; it lies in the V.P., and is really its vertical trace.

To obtain the shape of this section assume one part removed and a view taken at 90° to the plane EF. Hence construct an "auxiliary plan" on any plane parallel to EF, or, if the space be limited, it might be constructed on EF as a base.

Such a section is constructed on X_1Y_1, projections being made at 90° to EF from points where edges are obviously cut by the inclined plane, and plan distances of points in the section measured beyond X_1Y_1. We must notice here a most important point in the construction of sections, viz., that, as the section is treated as an auxiliary plan, we can only construct

it from a pair of projections—plan and elevation—as in previous cases where a complete solid was represented.

It therefore follows, that if a section plane such as EF shows in elevation the *edge* of the cut surface, we must construct the *plan* of the section before we can proceed with the auxiliary view.

The points in which the edge of the plane EF cuts the various lines in the elevation have their plans found directly by vertical projection on to the corresponding lines in plan. These, joined to form the plan outline of the section, give an idea of the shape, but a fore-shortened view. Notice that the measurements across this plan at 90° to XY are *true measurements*, but those parallel to XY are *not* true. The latter become lengthened to their correct value by the projection from the inclined cut in elevation.

The true shape of this section, then, is found by

(*a*) constructing a "plan" of the section, whose elevation is EF,

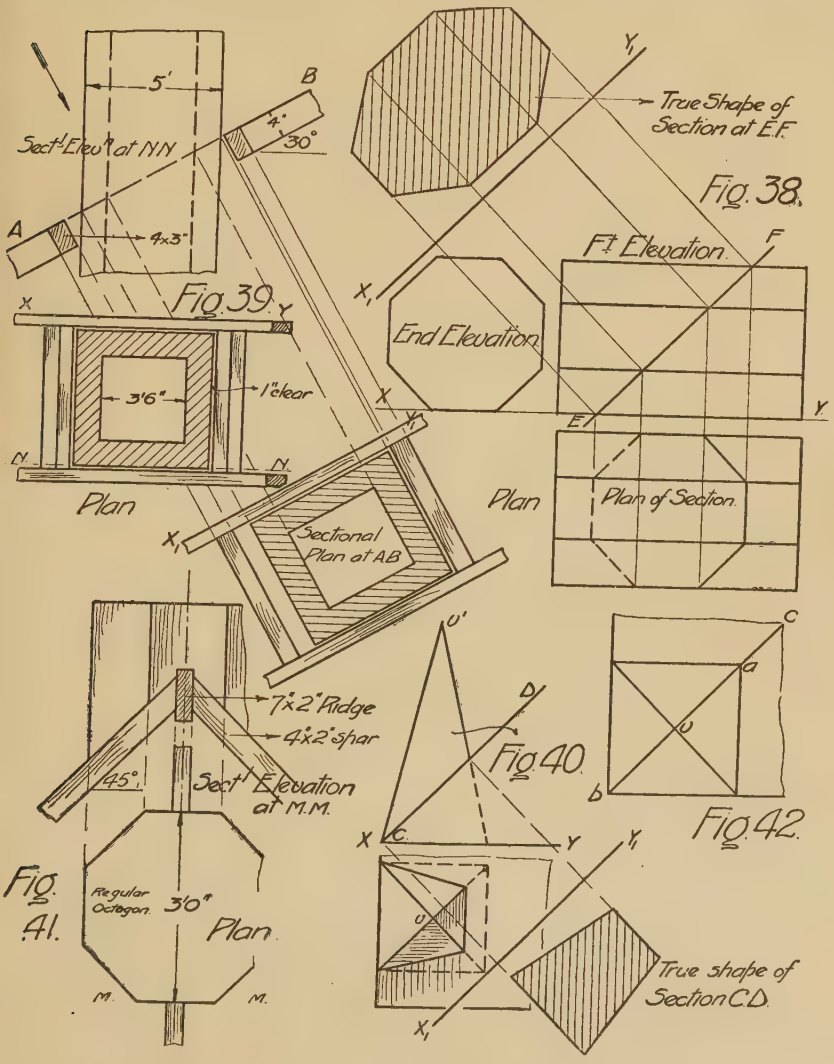
(*b*) projecting a "new" plan of the cut surface from the plan and elevation then available.

In building examples we find many cases of the intersection of solids by inclined planes. The larger number of these occur in roof construction. Take, as an instance, Fig. 39. Here, a square chimney rises through a roof, sloping at 30°, and the rafters are shown trimmed round the opening with a clearance of at least 1 in. The horizontal trimmers are placed at 90° to the slope, and are therefore only close to the brickwork at one edge.

Consider the roof surface AB as a plane intersecting the chimney. Find the true shape of the trimming round the stack, and show the section of the chimney by the roof. An auxiliary plan of the section at AB accomplishes this when taken at 90° to the roof plane (*see* arrow). By showing the form of the rafters *below* AB we convert our drawing into a "sectional plan." Work out this example fully, then try trimmings for other forms of shaft.

Fig. 40 illustrates the intersection of a pyramidal roof with a plane roof whose edge view is CD. The plan and elevation of the roofs are shown, and from the elevation it is clear that the roof plane CD cuts through the base of the pyramid at C, and through the back edges where the dotted line commences. As the angles or hips of the pyramid are visible in both views, the plans of the points of intersection are determinable by vertical projection, and straight lines joining these points complete the plan outline. *Observe that plane surfaces can only intersect each other in "straight lines."* Plan and elevation of

the cut surface being determined, the true shape is obtained by an auxiliary plan of the section. This figure constitutes



the form of the trimming, or framing, to the rafters at the intersection of the roofs.

The following examples, conscientiously worked, will do much to make you conversant with the process of projecting sections made by inclined planes. Some of these are of great

practical use when considered as intersections of structural solids. In their present form, the value may not be so apparent to the student; he will realise this, however, in later work.

EXAMPLES

(On sections by inclined planes)

(1) Consider an octagonal shaft of 2 ft. 3 ins. diameter passing vertically through a roof whose slope is 45°. Find the shape of the intersection of roof and shaft.

(2) Fig. 41 represents an octagonal ventilator base at the ridge of a pitched roof. Obtain the shape of the intersection of the base with the roof surface on one side of the ridge.

(3) Refer to Fig. 42. It is the plan of the angle of a hipped roof, whose slope is 30°, through which rises a pyramidal roof, whose base is a square of 6 ft. side, and vertical height 18 ft. The hips of the roof BC, and of the pyramid at BV and VA, coincide in plan, forming one straight line. Determine the plan of the line of intersection of these roofs by the aid of auxiliary projection. Add an elevation of the roofs. Make a freehand sketch of the angle.

(4) A block of stone has three inclined faces and three chamfered edges. One face is vertical (*see* Fig. 43). Required, the true shape of the sections SS and TT, which are at 90° to V.P.

The preliminary work to the section at SS is shown, the plan of the cut surface having been added in faint outline.

From these views project the true shape. Obtain section TT by similar procedure.

(5) A thick wall, battered on one face, is pierced by an opening, as illustrated in Fig. 44. The sectional elevation through the centre is shown by the left-hand drawing, and this is intersected by a plane, VV. Construct (*a*) the elevation of the section; (*b*) the true shape of the same.

(6) In Fig. 45 is represented the plan and elevation of an L-shaped stone base, "battered" on all faces uniformly. Let this block be required to spring from the inclined plane, UU; determine the real shape of the intersection at the springing surface.

(7) Fig. 46. An octagonal gate pier springs from a square base, the conversion of form being accomplished by chamfering or splaying the angles of the base block. Above this joint the horizontal section everywhere is a regular octagon. Construct the true shape of the two sections made by inclined planes, WW and ZZ, which cut at 90° to the V.P.

(8) A wooden hopper has the form and dimensions of Fig. 47. Draw its plan and elevation when made of 1½-in. material, and construct the true shape of the section at DD (whose plan is indicated in faint stroke and dot lines).

If the hopper be inverted and cut by a similar plane, it may then be considered as a hopper projecting through a roof, e.g., *a ventilator base, etc. The section would then become the trimming outline.*

Problems innumerable are met in regular building practice where the construction is dependent on the solution of similar problems to the above, and, although the practical case is often

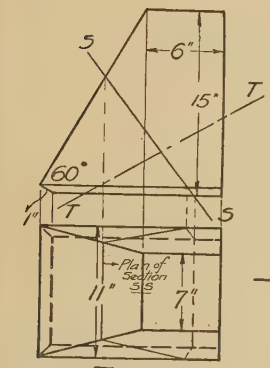


Fig 43

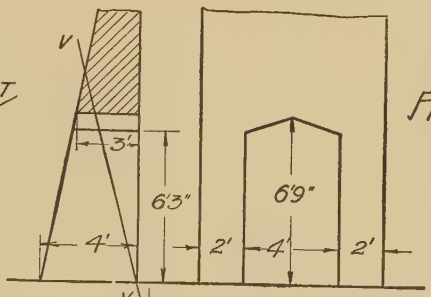


Fig 44

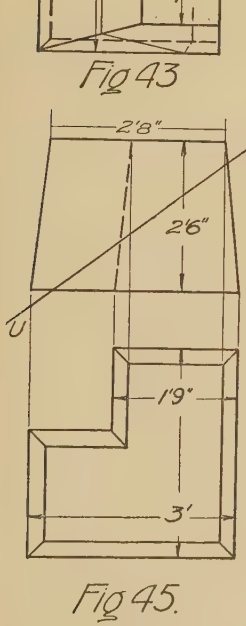


Fig 45

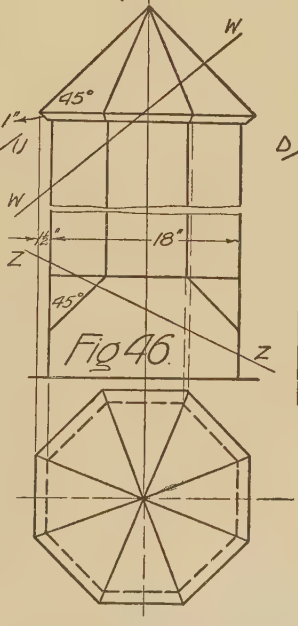


Fig 46

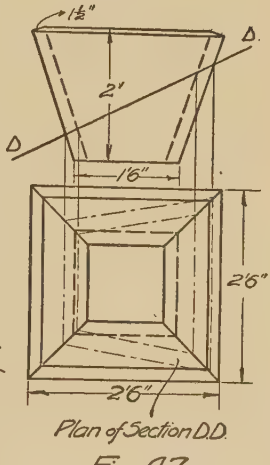


Fig 47

solved without the use of drawing paper, wherever and however the process of solution may be carried out it is always dependent on similar principles. You will realise this in later chapters where variations of "procedure" are introduced.

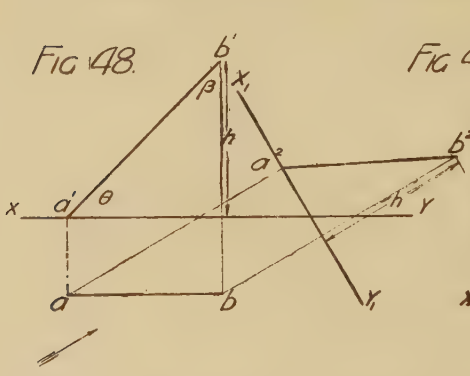
CHAPTER IV

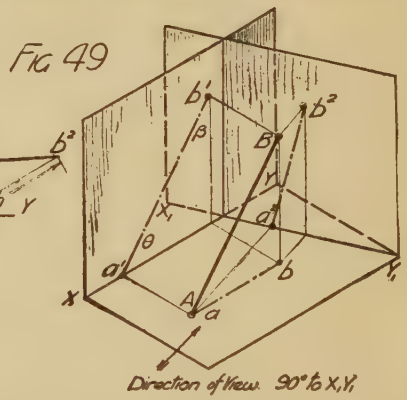
THE OBLIQUE LINE

HAVING given sufficient consideration to the projection of solids and sections thereof, we shall now commence the study of problems connected with the straight line in space, devoting special attention to the "oblique line."

We have previously seen, that when a line is *parallel* to one of the planes of projection, either H.P. or V.P., the *projection of the line on that plane exhibits its "true length" and its "inclination to the other plane."*

Such a line—parallel to the V.P., but inclined to the H.P.—is represented in Fig 48. The *plan* is parallel to V.P., showing





that the *line* is parallel to that plane, and the *elevation* is inclined at θ to the H.P., hence the *line* has that inclination.

Let a vertical line be dropped from the end B, then the inclined line AB makes an angle β (beta) with this vertical. (This is an *important angle*, being in most cases the bevel for the top vertical cut for a rafter. *See* vol. i. p. 110.)

Conceive other positions of an inclined line for yourself, and sketch its projections before following up Fig. 48 as described below.

We must now conceive the conditions under which a line

becomes "oblique," viz., *its position when it is inclined to both planes of projection.*

Since the line is said to be "an inclined line" so long as it remains parallel to one plane only, if it be moved from this position while maintaining its inclination to the other plane, it becomes an "oblique" line.

Instead of moving the *line* we may move the *plane*, and project a new elevation of the line on the latter. (The term "move the plane" is equivalent to saying "insert a *new* plane.")

Refer to Figs. 48 and 49. Suppose we decide to view the line in the direction of the arrow, a plane must be inserted behind the line at 90° to the direction of view and an auxiliary elevation of the line projected.

This process has been well developed in the preceding chapters and requires no special explanation. To assist the conception a sketch of the "inclined line" converted by a second elevation into an "oblique line" is given in Fig. 49.

In Fig. 48 the new elevation is made on X_1Y_1 by projecting from the plan at 90° to the new ground line and measuring the heights from the elevation. The new elevation of the line is marked a^2b^2. Now think of the plan and elevation in their relation to X_1Y_1 *only*, and you have a fairly common position of the oblique line represented. Suppose the paper to be revolved until X_1Y_1 takes up a horizontal position like the common ground line, then a better grasp of the relationship of the views is probably obtained. Do this with the book diagram and study it.

The consideration of Figs. 48 and 49 leads us to the solution of the oblique line. *Given* such an oblique line the solution required is generally the "true length," "inclination to the horizontal plane," and the "slope to a vertical line."

We may therefore convert the "oblique line" into an "inclined" one by the reverse process to problem 48, viz., by placing a vertical plane parallel to the *plan* of the line and projecting an auxiliary elevation upon it.

This reversed process is applied in Fig. 50, where an oblique line CD is shown in plan and elevation at cd, c^1d^1, and the true length (c^2a^2), the slope to H.P. (θ), and to a vertical line (β), are all determined by an elevation on a vertical plane parallel to the plan; c^1 and c^2 are on the ground plane in the two elevations, and d^2 is the same height above X_1Y_1 that d^1 is above XY.

The construction of the auxiliary elevation as employed here may be shortened by assuming that the *auxiliary vertical plane passes "through" the plan of the line*, thereby *containing* it. The plane is then turned about its XY (which is the plan of the line). Fig. 51 shows the result of this shortened process,

the plan only being reproduced from Fig. 50. The process is called *rabatment*, and is equivalent to rotating the right-

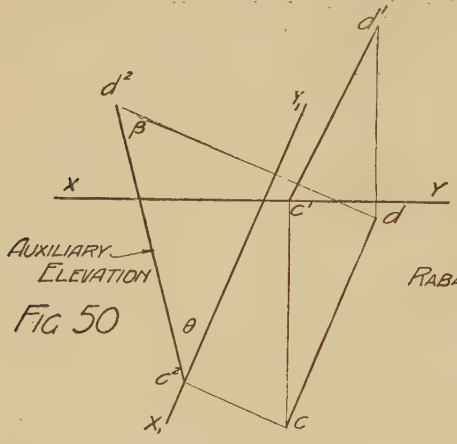


FIG. 50

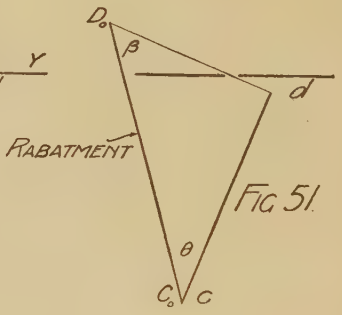
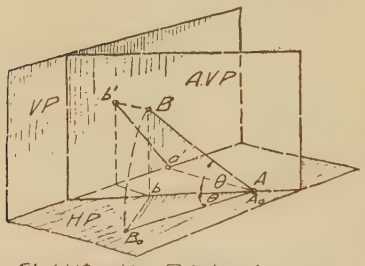


FIG. 51

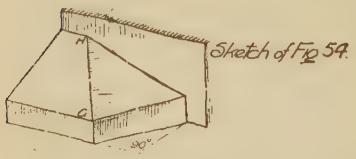
angled triangle (formed by the line CD, its plan *cd* and its perpendicular projector D*d*) into the H.P.

Fig. 52 shows another oblique line in the angle between V.P. and H.P. with a second vertical plane (A.V.P.) placed



Sketch to explain Rabatment

FIG. 52.



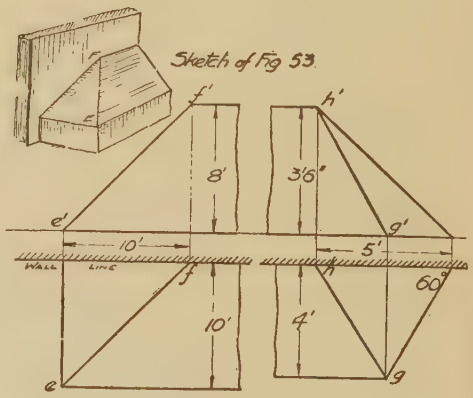


FIG. 53 FIG. 54

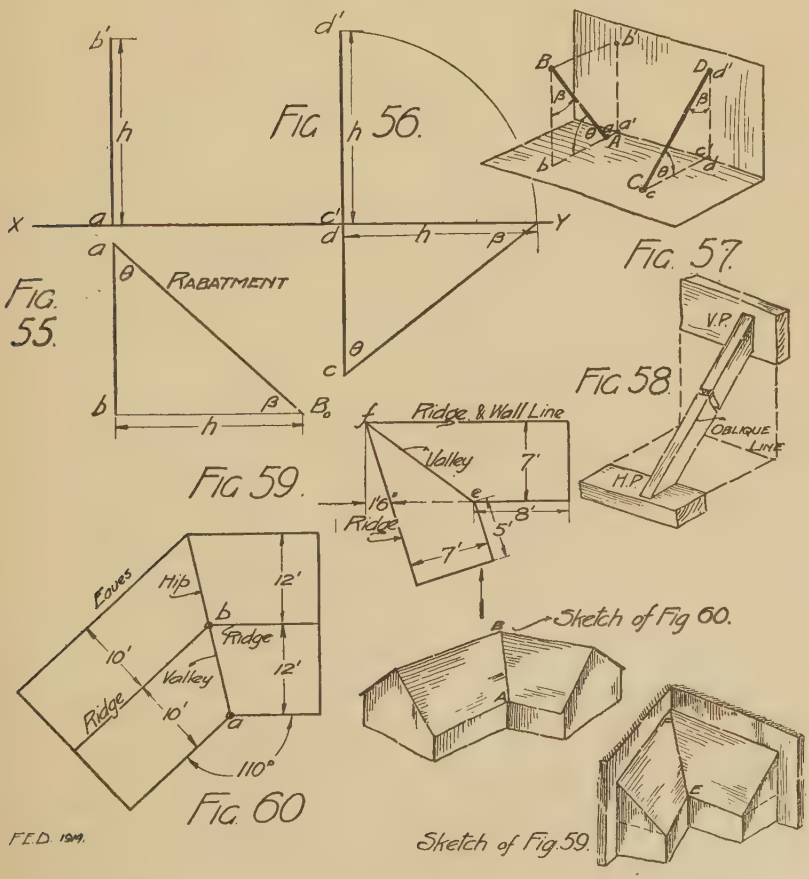
through the line AB. If the whole plane (A.V.P.) be turned about its base forwards into the H.P. the line goes with it and reaches the ground as shown at A_0B_0. Conceive the triangle A*b*B to move instead of the whole plane and you have the

idea of rabatment, which produces the same result. *Rabatment is a shortened method of auxiliary projection.*

Our next consideration is the recognition of similar oblique lines in practical cases. Two such lines are shown in Figs. 53 and 54, both of which are accompanied by explanatory sketches.

It would be wise to work three or four of the examples at the end of this chapter before proceeding further. Examples 1 to 4 illustrate the preceding problems.

A special position of the oblique line is dealt with in Figs. 55, 56, and 57. These figures illustrate what is most



probably the simplest position which an oblique line can occupy. In each case it is contained in a secondary vertical

plane, so that its plan and elevation are both at 90° to XY; thus, neither projection shows its true length.

Fig. 57 gives sketches of the two cases, and makes clear a matter which will arise in the further consideration of the previous cases of oblique lines. In these cases only (Fig. 57) the angle between the "oblique line" and a "vertical line" happens to coincide with the inclination to the V.P. (a problem not yet mentioned, but to be considered immediately). A side elevation, or rabatment, of the line is seen to solve the true length and inclinations as illustrated in Figs. 55 and 56.

Fig. 58 illustrates a *common rafter*, which, when placed as shown, has "oblique line" edges. The side elevation makes clear the angles for cutting against the wall plate (H.P.), and the ridge (V.P.). It should be observed here, that when such a rafter is represented with the main elevation showing the *side* of the rafter it is equivalent to a simple inclined line, and must be considered as such.

Refer now to the sketch attached to Fig. 61. AB is an oblique line similar to the line of Fig. 50. We have to learn how to conceive its inclination to the V.P.

The inclination of *any* line to a plane is obtained by placing a plane *through* the line and at 90° *to the plane to which the inclination is required, and rabatting the containing plane into it.* (This is the process we arrived at for the slope to H.P., and illustrated at Fig. 52.)

In Fig. 61 I.P. is an inclined plane *containing* the line, and is at 90° to V.P. Its *trace* on the V.P. coincides with the elevation of the line a^1b^1. The *angle between the "line and its elevation" is the inclination to V.P.*; this should be followed from the sketch of Fig. 61 before proceeding. Careful study is necessary here because the vital point is not easy to grasp. You must first realise that you need the true shape of the figure which lies in I.P. between the line AB and the V.P.

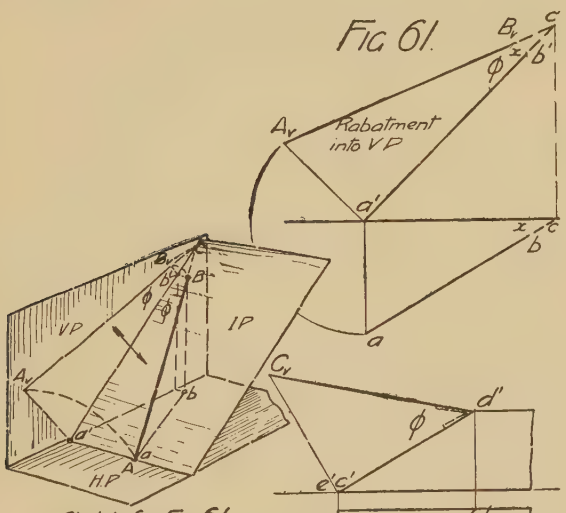
Having gripped the point to be solved it should then be clear that the true shape of this triangle a^1Ca may be obtained by an auxiliary plan in the direction of the arrow, or by direct rabatment of it, into the V.P. as indicated in the sketch.

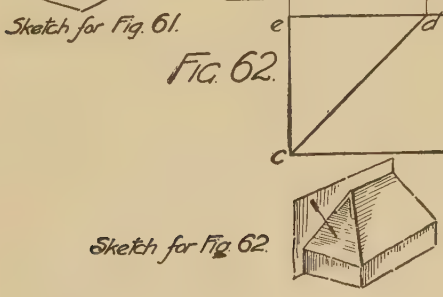
Refer to Fig. 61 where the rabatment is carried out in diagram form, and from which we will summarise the process. The plan and elevation of the line are ab and a^1b^1. To find true length and inclination to V.P. rabat the line into the vertical plane, which, by virtue of the process shown in sketch becomes—

Project from a^1b^1 and at 90° to it, measure on these projectors (from a^1b^1) the distances of a and b respectively in front

of the XY. The result is the line A_vB_v which, produced to C, encloses the required angle (ϕ), while A_vB_v is the true length.

In hip and valley roofs this angle is of great importance. An illustration of its value is shown in Fig. 62 and its accompanying sketch. The inclination of the hip to the wall





Sketch for Fig. 62.

is the angle at which the common rafters are cut on the *top surface* where these abut on the side of the hip. Such rafters are placed at 90° to the eaves line in plan, and the wall in this case is parallel to the rafters.

Fig. 62 (diagram) determines the slope of the hip CD to the wall plane—the angle between CD and DE.

Before completing the examples appended, refer back to Fig. 57 (and its diagrams 55 and 56), and make sure that you understood the statement that "in this case of the oblique line, the inclination of the line to the V.P. and to a vertical line coincide."

EXAMPLES

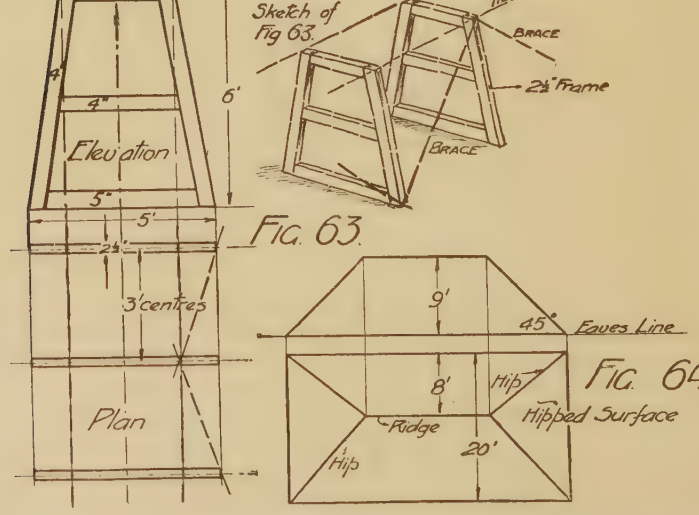
(1) Fig. 53 shows a lean-to roof with a hipped end. The intersection of the two roofs (*see* sketch) produces the oblique line, EF. Construct the plan and elevation to the particulars given, then determine—

(*a*) The true length of the hip ;
(*b*) The inclination to the H.P. ;
(*c*) „ a vertical line.

Refer to vol. i., pp. 110 and 111, and, comparing the hips of the two cases, state the use of each of your solutions, *a*, *b*, and *c*.

(2) Half of a bay window roof is shown in Fig. 54 and its adjoining sketch. The two roof surfaces make an obtuse angle at the eaves in plan, but the intersection, or hip, HG, is a similar oblique line to EF in the previous figure. Obtain the similar solutions, *a*, *b*, and *c*, to those of the first example.

(3) Fig. 59 is the plan of a conservatory, having a plan angle less than 90° between the two wings, and causing an intersecting roof along the "valley," FE. Reproduce the plan to scale, then, given that the ridge rises 5 ft. 6 ins. above the eaves, obtain the elevation in the direction of the arrow. Mark e^1f^1, and solve this line for length, and slopes to V.P. and a vertical line.



(4) Two V roofs intersect at an angle of 110° in plan (Fig. 60). Their spans vary, but the ridges are the same height in each roof, and situated in the centre of the plan. Hence the intersection occurs along a straight

line in plan, forming the "hip" and "valley" shown. Study this plan along with the freehand sketch. Let the height of the ridge be 9 ft. above the eaves, and construct an elevation in any direction you please. Solve the line AB under the three heads of Example I.

(5) Fig. 63 illustrates a series of vertical frames with sloping edges, which are to be used to carry a platform, formed of longitudinal bearers and cross boarding. These frames are to be braced diagonally in opposite directions, as shown in the sketch to the right. From the given particulars obtain the extreme length of the braces at the centre lines.

(6) The projections of a hipped roof of an irregular form are given at Fig. 64. For some special reason the ridge is placed 2 ft. out of the centre of the plan. Find the lengths and slopes of the different hips, so far as you know them to be required.

(7) Take a similar case to Fig. 62, using Fig. 53 for the purpose. Determine the inclination of the hip to the V.P., and explain the use of this angle.

NOTE

A matter of great importance arises out of the foregoing problems.

Although we have chosen to construct elevations of these roofs to given conditions, *this is unnecessary for the mere solution of the oblique lines* we have been considering.

Recall the process of rabatment. We may rabat any line into H.P. if we know the heights of its ends above the horizontal plane. In a roof it is usual to consider that a horizontal plane passes through and contains the eaves lines, and if the heights of required points above the eaves be given, an elevation may be dispensed with.

In Examples 3 and 4 above, as no elevation was given, yet the height of B above A, or F above E was known, the rabatment of these lines about their plans was at once possible. The solution in such a case is, therefore, rapidly obtained, and, if the only object in view is to *solve the line*, you should discard all extraneous labour and solve the line *only*.

The elevations asked for in the above problems serve other purposes. They make sure that the student has grasped the shape of the *whole roof*, and test his capacity to represent it from any point of view, exactly like any other geometrical solid. He also gets an excellent idea of the appearance of the roof, and should note whether the mass is pleasing or objectionable. (We are aware that "perspective" of the mass is the matter of great importance, but ordinary projection often points out primary defects.)

Before entering on the following chapters the student is advised to try the representation of planes and lines in approximate isometric or pictorial projection, in order to help his vision in attacking new problems where he fails to grip the "form" and "conditions."

In studying roof problems the conditions may be varied so very largely that models will probably not be available for a sufficient number of cases.

Paper models may be made by the student with great advantage, but they take time, and in advanced work the sketch may be made to serve the same purpose—with limitations, of course.

CHAPTER V

THE OBLIQUE PLANE

THE oblique plane presents to the student some of the greatest difficulties to be met with in descriptive geometry, and, when its study is unaccompanied by practical application, it may become very tedious.

In *your* case, you have the greatest possible incentive to study problems arising from its practical application, because they form the basis of so much work in connection with hand-railing, roofing, and special constructions of any kind where intersections of surfaces may occur.

You have had several opportunities to become conversant with the inclined plane, and we shall now need to use it as a basis of comparison with other planes of "double" inclination.

In order to compare the positions of different types of plane a series of sketches representing one of each common type, with the method of measuring its inclinations to H.P. and V.P., are shown in Fig. 65.

Fig. 65 contains seven sketches lettered A to G. Fig. A shows the simple inclined plane whose edge view in front elevation represents the slope to H.P. Fig. B shows the inclined plane having a slope to V.P. only, because it is itself vertical. The edge view is seen in plan, and exhibits the inclination to V.P.

NOTE.—A and B have been considered frequently in connection with auxiliary projection, and need little reference beyond the statement that these, like all other planes, may be completely represented by their traces —lines of intersection with the V.P. and H.P.—which, when the planes are opened out, appear as *lines*, and are referred to as vertical trace (V.T.), and horizontal trace (H.T.). The position of these traces explain the position of the plane at a glance, for, if one of them is at 90° to XY, the plane it represents is at 90° to the opposite plane.

If the traces are *both* inclined to XY (or in one special case parallel to it), we know at once that the plane it represents is inclined to both planes of projection.

Fig. C shows a plane which, though *oblique*, may be thought

of as a *side elevation of case A.* It is inclined to both H.P. and V.P., but its *edge view shows the slope*; therefore, if a

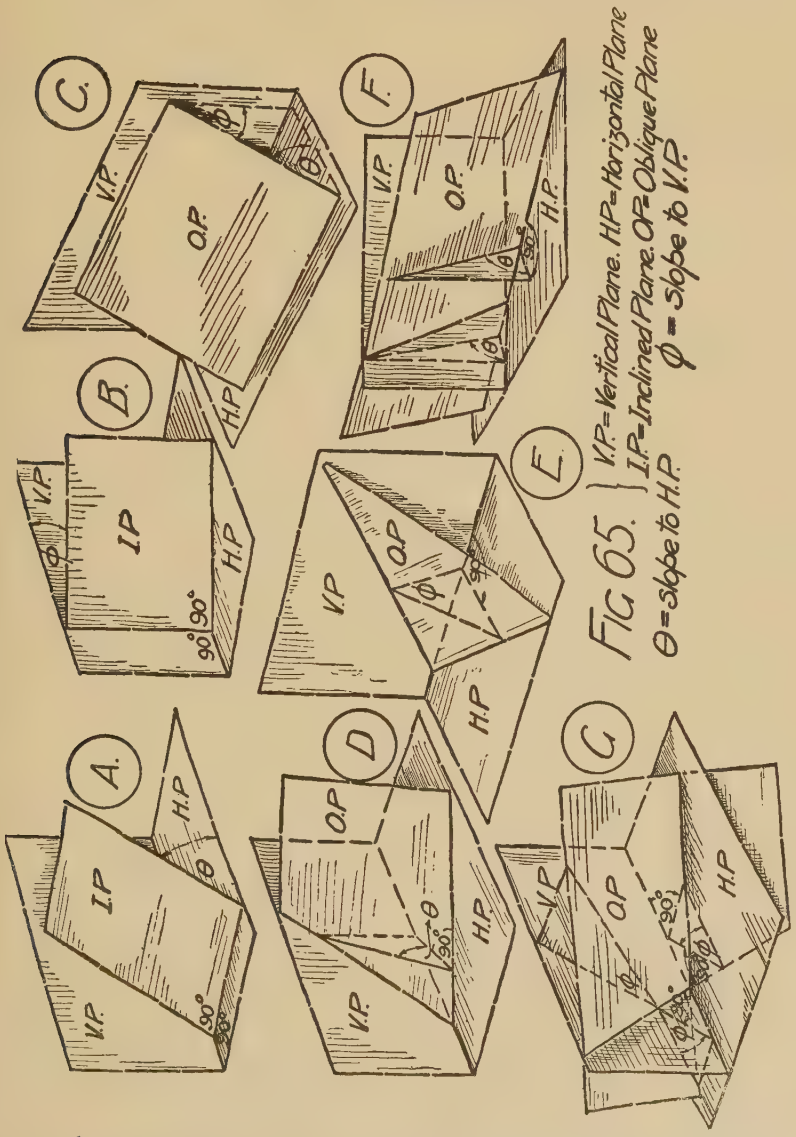


FIG 65.

secondary elevation be drawn (or a sectional elevation at 90° to both planes), the slopes θ and ϕ are truly represented as indicated on the "end" of the figure.

Fig. D illustrates the case where the plane has its traces sloping away to the right from the point of intersection on the ground line; the angle between the cutting edges is *less* than a right angle. To measure its slope to the ground plane we must determine the shape of the vertical triangle whose plane is at 90° to the H.T.

Fig. E depicts the same oblique plane, viewed from the opposite side, with the triangle indicated for measuring the slope to the vertical plane. The plane of this triangle is at 90° to the V.T.

Fig. F shows an oblique plane sloping *forwards* (instead of backwards as in the previous case), and the vertical trace is seen to slope *upwards to the left*. The triangle for measuring the slope to the horizontal plane is placed in two available positions, viz., in front of the V.P. and behind it. The latter position is more commonly used, as will be seen later. Both triangles stand vertically, and at 90° to the H.T.

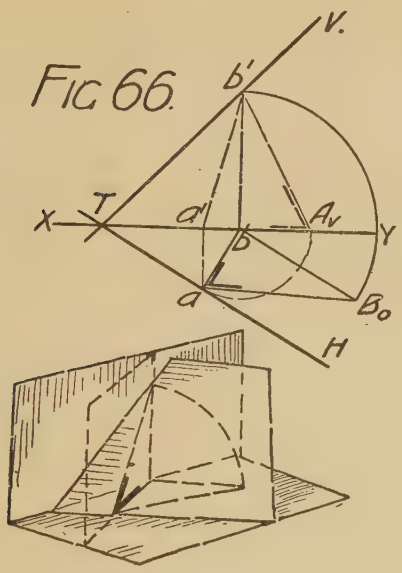
Fig. G indicates the same oblique plane as Fig. F with the method of measuring the slope to V.P. by triangles inserted between the oblique plane and the V.P., and at 90° to the vertical trace. Three positions of similar triangles are indicated; they are obviously parallel to each other.

NOTE.—The various sketches of Fig. 65 are intended to be studied with a view to familiarising the student with the *positions of the planes* and the meaning of the *inclination of one plane to another*. Then the problems following may be studied individually, and in full detail, as representing the more important features of these planes so far as we may deal with them at this stage.

Fig. 66 and the attached sketch reproduces the conditions of Fig. 65 D, and further explains the process of constructing and measuring the inclination of the oblique plane VTH to the horizontal. Referring to the freehand sketch, if, at any position along the H.T., we place a vertical plane at 90° to the trace of the oblique plane (*see* dotted outline), a portion of it is severed by and enclosed within the three planes, forming the vertical right-angled triangle shown. The right angle is at the XY, and the base angle (∠) measures the inclination required. While in its real position, the plan of the triangle is *ab* and the elevation a^1b^1b (Fig. 66), and its true shape is not visible in either view, hence we must draw an auxiliary elevation, or a rabatment of the triangle.

The sketch shows the triangle rabatted about its base into the H.P., *where a plan would then exhibit its true shape.*

Fig 66 shows this construction. The traces of the given plane are drawn as they would appear on the opened planes, and each trace continued a little beyond XY to indicate that the plane is unlimited in extent and may pass *behind* the V.P. and *below* the H.P. if required. Across the plan, at 90° to H.T., the edge of the vertical cutting plane is shown



Sketch of Fig. 66.

at *ab*, and in elevation the intersection of this plane with the V.P. is shown at bb^1; a^1b^1 is, therefore, the elevation of the hypotenuse of the triangle which is formed beneath the plane.

As this triangle is now completely represented in plan and elevation, rabat it into the ground by laying out the height bb^1 at bB_0 using the compasses to transfer the height and indicate from what line the measurement has been made.

Important.—Because the right-angled triangle may always be constructed from a known base and height, we may now shorten the whole

procedure to the following, assuming the aim to be clearly understood.

Method.—Take any point *a* on H.T., draw *ab* at 90° to cut XY. Erect a perpendicular at *b* to cut V.T. in b^1. Draw bB_0 at 90° to *ab*, making its length equal to bb^1. Join B_0 to *a*. Then baB_0 is the angle required.

The dotted triangle b^1A_0b represents another solution of the same problem, where the triangle is rotated about its perpendicular into the V.P. You will recognise that the resulting triangle is the same, for it has the same base and perpendicular lengths assembled at the right angle. You may build the triangle where and how you like, so long as you recognise that these lengths (base and perpendicular) are the vital elements. In practical work they are very often measured and set up about the right angle without any further reference or connection.

On drawing paper it is wise to adopt one method of working and *generally* adhere to it under ordinary conditions.

If you still have difficulty in grasping this initial problem make a stiff paper or cardboard model of the planes. This may be done as follows: By the method indicated pictorially at Fig. 67 lay out the oblique plane on the H.P. by supposing it to be revolved about its horizontal trace outwards. This involves finding the correct position of the line TB_H, and depends on the position of point B when revolved with the plane. Its path during revolution would be the arc BB_H, and this would appear in plan as a straight line; in the plan we can therefore draw the path at 90° through *a*, making it a continuation of *ba*—B_H evidently lies upon it, and is distant TB from the angle T. The length TB_H may therefore be measured from T to cut the path AB_H at the latter point.

Now refer to the projections of this problem at Fig. 68, which must be reproduced on cardboard or stiff paper. Draw the traces VTH. Choose any point *a* in H.T.; place *ba* at right angles and produce outwards from XY. Erect bb^1 perpendicular to XY. Take T as centre and radius Tb^1, and transfer this to intersect *ba* produced at B_H. Join TB_H and produce a little to C. Make CD and DH any irregular boundary to limit the plane for your purpose.

Now rabat the triangle for slope to H.P. at baB_0, as per Fig. 66. Limit the V.P. by boundaries XE, EF, FY. Then sever along the lines marked "cut," lift up the vertical plane at right angles and fold the developed oblique plane (TCDH)

against it. It should meet at the trace V.T. Erect the triangle underneath to stand vertically above *ab*, and if your work has been correctly done the triangle $ab B_0$ will fit accurately against the plane.

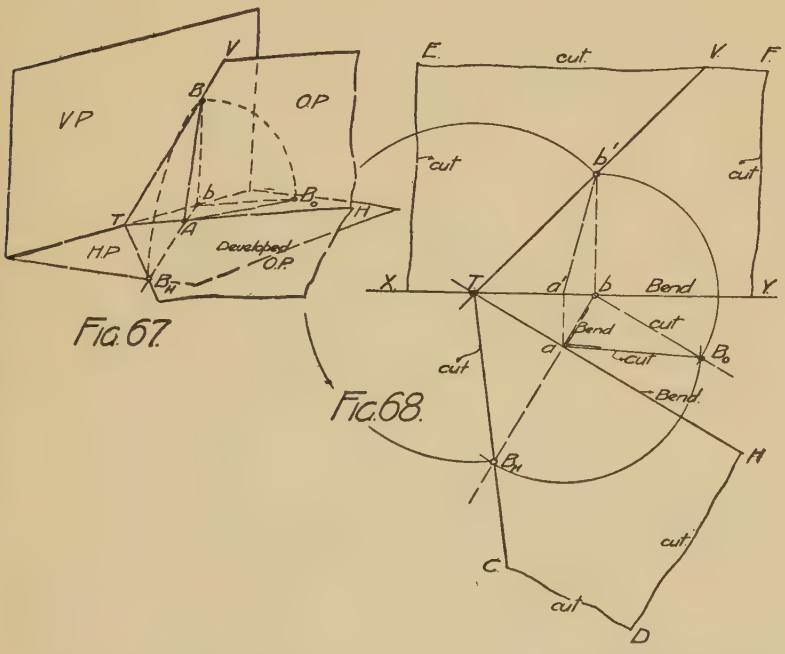


Fig. 67.

Fig. 68.

You should follow this with a model to demonstrate the angle between the oblique plane and the *vertical* plane using Fig. 69, and sketch for your basis (*see* clauses immediately following).

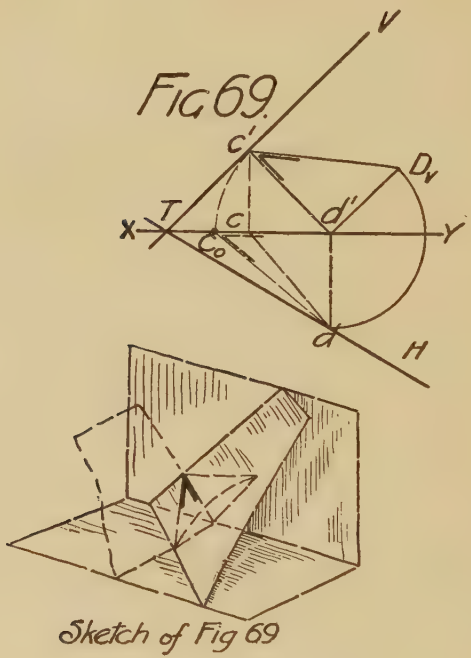
Other models are easily made for similar demonstrations *where necessary.*

We must now concentrate on determining the slope of the same oblique plane to the vertical plane. The general method has already been indicated at Fig. 65 E.

Fig. 69 and its explanatory sketch show the detailed method of procedure. In the sketch a dotted plane has been inserted at 90° both to the V.T. of the oblique plane and to the vertical plane. (The sharp student will have found ere this, that when a plane is at 90° to a *trace* it is also at 90° to *both the planes "forming" the trace.*)

A portion of the dotted plane in the figure is intercepted by the oblique plane, etc., forming a right-angled triangle at XY

whose vertical angle (emphasised by the thick lines) is the angle between the planes. As this triangle does not exhibit its true shape in plan and elevation, it must, as in previous cases, be rabatted into a convenient plane for view. In our sketch the triangle has been rabatted about the edge intersecting V.P. until it lies in the vertical plane. An elevation of it in this position shows the true shape.



Sketch of Fig 69

Referring now to the orthogonal view (Fig. 69), the plan and elevation of the intercepted triangle are first obtained; c^1d^1 is the edge view of it, and the plan is cd^1d. Taking c^1d^1 as base, rabat the triangle by transferring the length a^1d to d^1D_v which is at 90° to c^1a^1. The angle $d^1c^1D_v$ is the inclination to the V.P.

The solution might have been obtained also by revolving the triangle cd^1d about its lower edge until the vertex c lies on ground plane at C_0; dC_0d^1 is the same angle as before.

Condensing our process as in problem 66 we have—

> To find the slope to the V.P. take any point c^1 on the V.T., drop a perpendicular from the trace, meeting XY at d^1; from d^1 draw at 90° to the XY intersecting HT at d. Take d^1 as centre and d^1d as radius and transfer to the perpendicular at d^1 D_v. Join $D_v c^1$, thus completing the right-angled triangle; the angle at c^1 is the inclination required.

We have another case to consider, whose application you will not often meet in regular practice. It *does* occur, and therefore cannot be passed over.

The sketch of Fig. 70 shows the position of the oblique plane, which is similar to case F, Fig. 65. Observe that the plane slopes forwards from the horizontal trace. The *method* of measuring the angle must of necessity be the same, viz., intersect the two given planes by a third plane at 90°, and find the included angle.

Observe that in Fig. 70 the oblique plane has been continued beyond the vertical plane, and that *the part "behind" the V.P. is identical in position with that of Figs.* 66 *and* 67. To realise this move your point of view to the opposite side of the planes.

We cannot conveniently alter the conditions of problems such as this, so as to convert them *all* into the easier cases, hence we shall accept the exact conditions, represent the plane by its traces, and proceed to the solution.

Determine, then, in Fig. 70, the slope to the H.P. Place a V.P. at 90° to the oblique plane (*see* sketch). A right-angled triangle is intercepted whose base $\angle$ is the inclination to H.P.; this is clearly marked in the sketch *behind* the V.P. To view this properly, rabat it into the H.P. about its base; the path of the vertex is shown by the arc.

The diagram of Fig. 70 shows this operation carried out orthogonally. Note, first, that the triangle we seek is *behind* the V.P., which causes our *working* to be *also* behind. Produce the H.T. backwards, and think of this trace as seen "wholly" from above while the V.T. is seen entirely in a front elevation. To construct the triangle of slope, take any point on the H.T. (behind), and draw at 90° from the trace to meet XY. Erect a perpendicular from XY in the vertical plane to meet the V.T. Join intersection with V.T. to the elevation of the first point on XY. Lay the triangle on its face in the H.P.

This process may be traced on the sketch and then worked out in detail at Fig. 70.

Following the geometrical working of Fig. 70, take a point *e* on the H.T. behind XY, draw perpendicular to meet

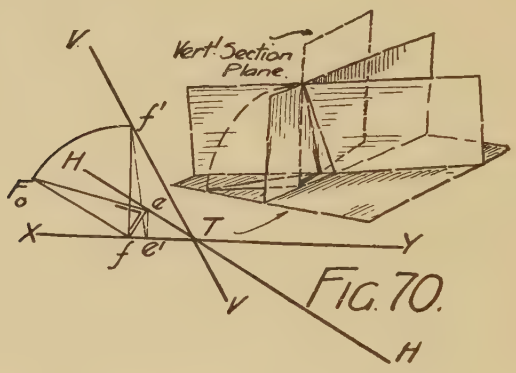


FIG. 70.

XY at *f*, from *f* erect a perpendicular to the XY intersecting the V.T. at f^1. Place *ef* and ff^1 at right angles in the rabatment, and complete the triangle $fe\mathrm{F}_0$. The angle at *e* is the inclination required.

To make the working clear the plan and elevation of this triangle are shown at *ef* and ff^1e^1 respectively, *ef* being the "edge view" in plan.

The slope of the same plane to V.P. is determined in Fig. 71 and its sketch. The sketch gives the right-angled triangle fitted between the oblique plane and the V.P. *underneath* the H.P., this being the most convenient position.

In the sketch follow the triangle from the black vertex which lies on the V.T. It is part of the dotted plane cutting the trace at right angles, and is the portion intercepted and enclosed by the other three planes. To view this triangle properly, rabat it about its perpendicular into V.P. Now follow the same process in the diagram of Fig. 71. Abbreviated, it is as follows :—

Take any point g^1 on the V.T. *below* XY, draw at 90° to V.T. intersecting XY at h^1. From h^1 drop a perpendicular to XY meeting H.T. in *h*. Project g^1 to XY obtaining the plan of the point *g*. Then g^1h^1 is the "edge view" of the triangle in elevation and hgh^1 is the "plan."

Draw $h^1\mathrm{H}_0$ at 90° to g^1h^1 and cut off h^1h along it at H_0;

then $h^1g^1H_0$ is the rabatted triangle, the angle at g^1 being the required slope.

Observe, that in all these instances of measuring the inclination of an oblique plane, one system of rabatting has been adopted, viz., *rabatting the triangle about the edge contained in "that" plane to which the inclination is required.* Hence, in the results we have obtained, the desired angle is, in every case,

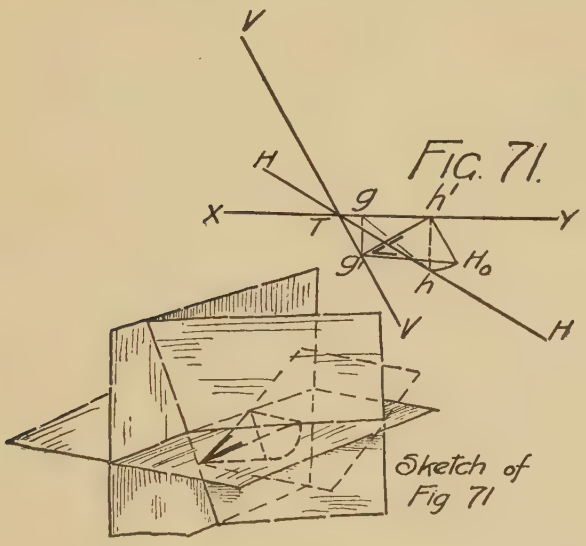


FIG. 71.

the angle nearest to the trace of the plane, on "that plane" to which slope is being "measured."

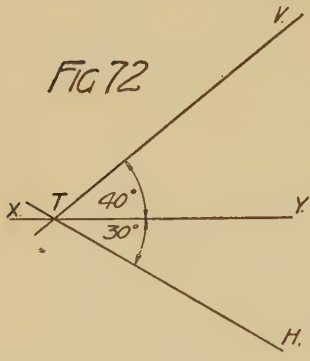
There are many other problems on the inclination of planes to each other, some of which are specially applicable to handrailing, but they are too numerous and complicated to include in this course of study. We must now obtain conversance with the foregoing principles by working through a selection of examples.

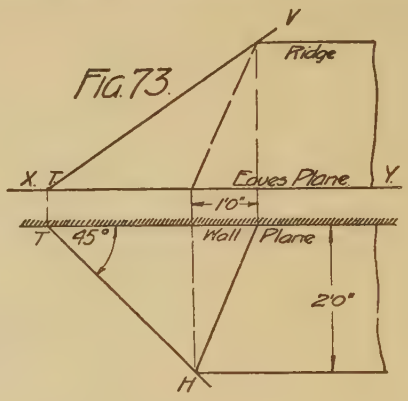
EXAMPLES

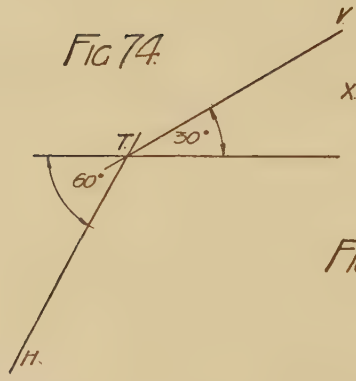
(1) In a hand-railing problem it was found that the traces of the oblique plane containing the sloping plank out of which the rail was to be cut had the position represented by Fig. 72. To place the section of the rail on the jointed end it is necessary to know the angle

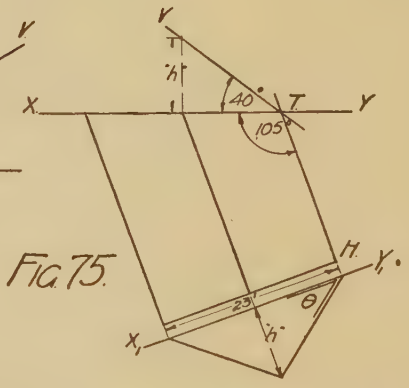
between the plane VTH and the vertical plane. Determine this, and mark it clearly on your paper.

(2) In Fig. 73 is shown a lean-to roof with a "return" or "hipped end" as in a bay-window. It is known that the ridge rises 1 ft. 9 ins. above









the eaves. Draw the two views, and determine the "slope to the horizontal" of the return roof, which is an oblique plane.

HINT.—So as to make this case to correspond exactly with previous work, place the plan of the wall face close to XY. The two points T will then coincide.

(3) Determine the slope of the given plane, Fig. 74, to V.P. and H.P. These views may be worked in one diagram or in two separate views to avoid confusion.

(4) Fig. 75 gives the plan of a vee roof abutting on an external wall of a larger warehouse building. A sectional elevation on X_1Y_1 will show the true slope of this roof. On the plane above XY (representing the warehouse wall) construct the trace of the given roof slope on the right. In order to confirm your principles for determining the slope

of an oblique plane to the horizontal, find the inclination of VTH to the H.P., and compare with the angle θ in the true section of the roof, which you are to determine from the elevation on XY.

(5) Apply the same methods and solve the slope to H.P. on the left-hand side of the roof, using the traces of the oblique plane for the purpose.

Note that Figs. 65 D and F, and Figs. 72 and 74 represent the types of oblique plane occurring in this roof example.

(6) The H.T. of an oblique plane makes 45° with XY. The oblique plane has a "real slope" of 30° to the horizontal. Determine the vertical trace.

HINT.—Work backwards on Fig. 66.

CHAPTER VI

CONVERSION OF THE OBLIQUE PLANE—SIMPLIFYING PROBLEMS

WE can often save much time and labour of thought by eliminating the oblique plane, as such, from our problems.

Let us concentrate for a time, then, on the constitution of the oblique plane, how it comes to exist, and how we may logically think of it as a simple inclined plane.

We have seen that any plane inserted in the angle between V.P. and H.P. is an oblique plane, if it does not show its edge view in either plan or elevation.

If its edge view is seen in *either* projection it has *one inclination only*. It therefore follows that an inclined plane may be converted into an oblique plane by inserting a new V.P. which is not at 90° to the plane.

Fig. 76 explains how this conversion takes place. V.P. (1) is the ordinary vertical plane, and the inclined plane is shown intersecting it at V.T., and making an angle θ with the horizontal plane.

Let the inclined plane be continued beyond the V.P. (1), and place a new vertical plane, V.P. (2), so as to be inclined to V.P. (1). The horizontal trace of the sloping plane is still in the same position, but prolonged to T. The vertical trace of the plane, however (while passing through the point A, which is common to the three planes intersecting there), has changed position, having become V_1T_1.

As viewed in its relationship to V.P. (2), *viz., at* 90° *to the latter, the "simple inclined plane" has now become an "oblique plane."*

It is obvious that in "converting" the plane, we have not altered its inclination to the horizontal, for it has not been moved; it should further be clear that to see its real inclination we need only to consider it in its first position, viewed at 90° to V.P. (1). From this consideration we derive our "principle

of conversion," which is the "reverse process" to the one illustrated in Figs. 76 to 79 inclusive.

In order to get a very clear grasp of the conditions, we shall go a little closer into these preliminary cases.

Figs. 76 and 77 show by sketch and diagram how the inclined plane is converted into an oblique plane by the insertion of an auxiliary V.P., so that the angle between the

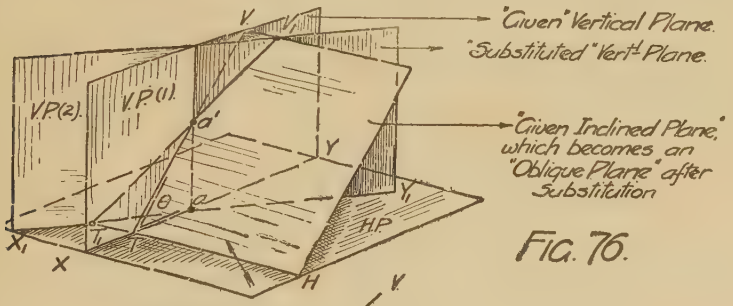


FIG. 76.

Sketch of Fig. 77

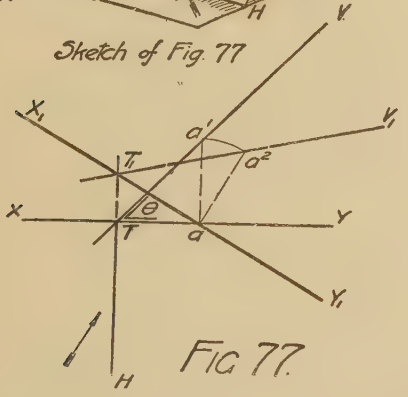


FIG 77.

traces of the resulting oblique plane is less than 90°. Thus, in Fig. 77 VTH represents the inclined plane; the new V.P. is represented by X_1Y_1, being its ground trace.

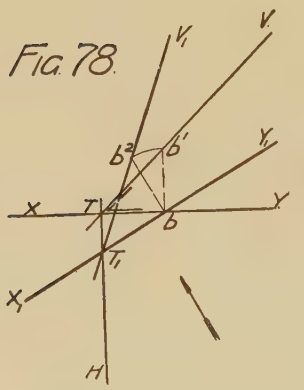
Observe that the horizontal trace, lying on the H.P., is produced until it meets X_1Y_1 on the ground behind V.P. at T_1. An elevation of the planes is now to be thought of, viewed in a direction at 90° to X_1Y_1, and we desire to know where the elevation of the vertical trace of the sloping plane will lie on the new V.P. — that is, above X_1Y_1. For this purpose, *we must find the height of a common point of intersection*, where the two vertical planes and the inclined plane meet.

The two V.P.'s meet in a vertical line whose plan is

represented by the point a, and, as the trace V.T. cuts through *both* vertical planes in this vertical line, the point, in a perpendicular projector from a, which lies in the V.T., must be the *common point of intersection*, viz., a^1. Hence, draw the auxiliary elevation of A on the rabatted plane above X_1Y_1, projecting from the plan a and measuring aa^1 at aa^2. Then a^2 is the elevation of a point in the new trace. Place V_1T_1 through a^2 and T_1, and the conversion is complete.

It is easily understood on reference to Fig. 76 (sketch), that the angle $V_1T_1H_1$ is less than 90°. This is often spoken of as the "angle between the traces."

The same process of converting an inclined plane into an oblique plane is shown in Fig. 79; but the new V.P. is made



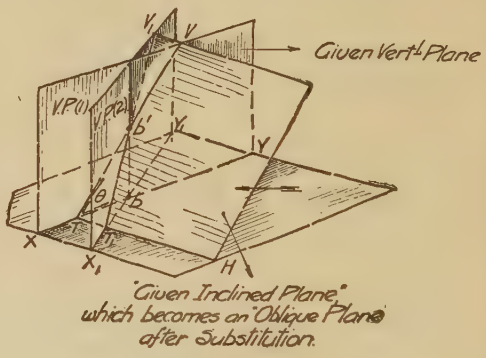


Fig. 79
Sketch of Fig. 78.

to slope in the opposite direction, cutting off a portion of the internal angle of the inclined plane. The result of this is to make the angle between the traces greater than 90°. In other respects the conditions of conversion are similar to those of Fig. 76, and the projections of Fig. 79 are worked in detail at Fig. 78.

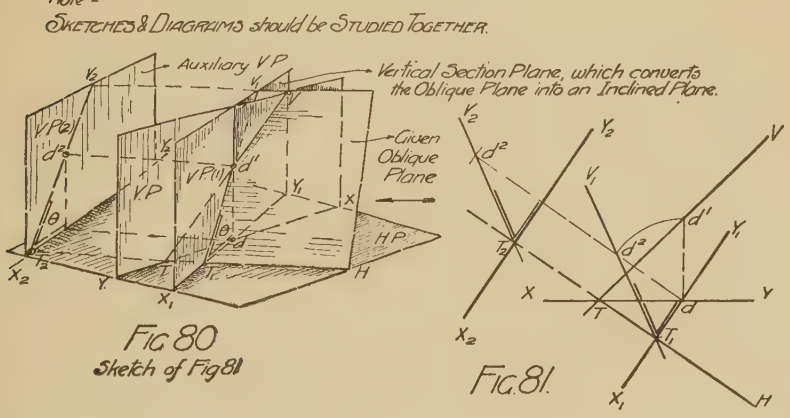
Study these figures in pairs, so that you grasp the process in the sketch before applying it orthogonally.

We may now establish the "principle of conversion" referred to in a previous clause, viz.:—

> As we may convert an inclined plane into an oblique plane by substituting an auxiliary V.P. at an inclination to the primary vertical plane, so *we may convert an "oblique plane" into an*

"inclined plane," by substituting an auxiliary V.P. at 90° to the horizontal trace of the oblique plane, and taking an "edge view" of it. See Figs. 80 and 81.

In Fig. 80 the converting plane V.P. (1) is shown cutting across the oblique plane. If this plane, with the new vertical trace upon it, be revolved about its X_1Y_1 into the ground plane, we have the result shown in Fig. 81. If for clearness of this diagram, we prefer to "isolate" the projection of the edge view of the plane, we may assume the new V.P. to be withdrawn backwards to the position X_2Y_2 as shown in Figs. 80 and 81 to the left. Observe that *the projections on V.P.* (1)



and V.P. (2) *are "identical," because the planes are "parallel."* (It is often an advantage to be able to isolate the converted plane from the original oblique traces, especially when other problems are to be based and worked upon it.)

Fig. 82 shows the conversion of an oblique plane occasionally arising, and worthy of inclusion here as a test of your grasp of the principle involved.

VTH is the given plane sloping upwards towards you when viewing the planes from the front. (It is identical with the plane of Figs. 70 and 71, whose slopes we have previously measured.)

Place a new V.P. at X_1Y_1, making 90° with H.T., intersecting it at T_1 and XY at *c*. C is the common point of intersection of the two V.P.'s and the oblique plane. Its height is determined by placing its elevation on the V.T. at c^1.

To convert the plane, construct an "edge elevation" on the

new V.P. by projecting from c at 90° to X_1Y_1 and measuring the height cc^1 at cc^2. Join c^2T_1 and produce, giving the new vertical trace, V_1T_1. Viewed in the direction of the arrow,

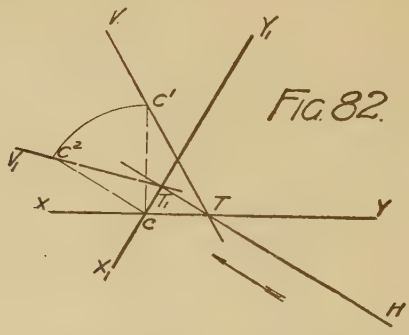


Fig. 82.

V_1T_1H is the converted plane, and $X_1T_1V_1$ is its inclination to the horizontal plane.

NOTE.—You have established another method, therefore, of measuring the slope of this plane to the H.P. Its basis is clear in Fig. 65 F, which you might wisely examine along with Fig. 82.

EXAMPLES

(1) Convert the return roof slope of the bay-window in Fig. 54 into an equivalent inclined plane. Determine and state its slope to H.P. from the converted plane.

(2) Obtain an equivalent inclined plane to the oblique plane at the hipped end of the lean-to roof in Fig. 73. What is the angle between the H.P. and the roof surface, and of what practical use is this angle?

(3) Refer to Fig. 75. Convert the given plane VTH into an inclined plane. If you have previously solved the problem attached to Fig. 75 (earlier examples) you will have an additional check on your result.

(4) Fig. 72 gives the traces of an oblique plane. Solve the equivalent inclined plane, placing its edge view in a detached position behind the diagram. Measure the slope to H.P., and see that it agrees with the slope previously obtained.

(5) The angle between the traces of an oblique plane is greater than 90°, so that the traces appear as in Fig. 74. Reproduce these. Determine the equivalent inclined plane; represent it clearly and measure its inclination to the ground plane.

(6) The horizontal trace of an oblique plane makes an angle of 36° with the XY. It has an inclination of 45° to the H.P. Construct its edge elevation, and from this determine the vertical traces of the oblique plane.

(7) Determine the angle between the traces of the oblique plane in problem 6, that is, obtain the real shape of that portion or angle of the plane contained between the V.P. and H.P. (Refer to the diagram in Fig. 68 if you have previously overlooked the principle of developing or rabatting it.)

(8) Now employ your results of Example 7 to the solution of the following problem. Given that the H.T. makes 45° with XY, and the angle between the traces is 60°, construct the traces of the plane. What slope has this plane to the H.P. and V.P. ?

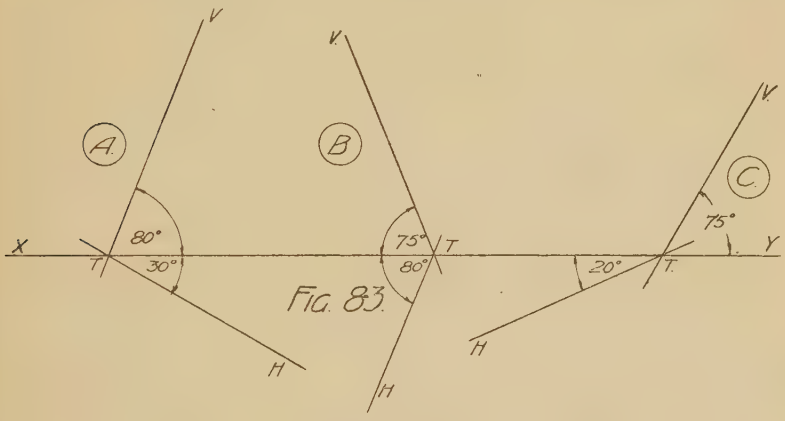


FIG. 83.

(9) Fig. 83 shows three cases (A, B, and C) of the oblique plane. Determine in each case—

(*a*) The inclined plane, by conversion ;
(*b*) The slope to H.P., from the conversion or by section plane ;
(*c*) The inclination to V.P. ;
(*d*) The angle between the traces.

NOTE.—*See* later chapters for the application of simplified oblique planes.

CHAPTER VII

GENERAL INTERSECTION OF PLANES

WHEN two planes cut each other, they do so in a "straight line," and form, or contain, a solid angle between the surfaces. This angle is called a "dihedral" angle.

In determining the slope of one plane to another, during our consideration of the inclined and oblique planes we have been measuring these contained angles, and our principle has been to "place a section plane at 90° to the given intersecting planes and determine the angle between the traces formed thereon." Reference to Figs. 66, 69, 70, and 71 will recall this principle.

(There is a possibility of this principle having been mechanically applied after proving one case, and as we desire to avoid mere *methods* we have restated the principle briefly, and shall apply it again to the more difficult problems immediately following.)

Our object now, is to develop the method of finding the dihedral angle between any pair of intersecting planes wherever they may be placed, and to apply the above principle in the solution.

Before we can consider the intersection of two planes such as roof surfaces or handrail planes, we must have, or be able to obtain by construction, all particulars of the line in which the intersection occurs.

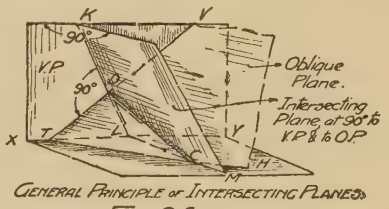
A plane is usually *represented* by its traces, though it may be stated by particulars enabling its traces to be found; *given* the traces, it is *fully known*.

Let two such planes be given, then their line of intersection may at once be constructed, for, being straight, it must lie between the point of intersection of the horizontal traces and the point of intersection of the vertical traces. Hence, if this line be determined we can at once apply the section plane at right angles to it, for, like any other trace, it lies wholly in both planes causing it.

In order to start you on the right line of thought Fig. 84 is reproduced from previous problems on the oblique plane,

and the method of finding the angle between the oblique and vertical planes made clear.

We have said that the "intersecting plane" must be at 90° to both of the planes containing the angle, and this condition is attained when the plane cuts the line of intersection at 90°. Hence the intersecting plane KLM is placed at 90° to V.T. and the angle between the planes is the angle contained between the lines LD and DC (or the supplementary angle KDC). These lines are the *traces* of the section plane KLM.



GENERAL PRINCIPLE OF INTERSECTING PLANES.

FIG. 84.

Fig. 85 shows this angle developed — rabatted into H.P., similar to previous solutions—at LD_0c.

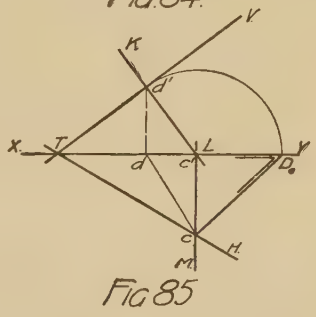


FIG 85

HIPS

Now let us consider a constructional case. To the left of Fig. 87 is the sketch of a small triangular bay, or oriel window roof with a V-hipped covering. We shall suppose that the roof is to be tiled, hip tiles (bent across the angle in one piece) being employed at the intersection. To make, or to select the hip tiles, we need to know what angle the two tile surfaces make with each other.

Compare the sketch with Fig. 86, which is the conventional arrangement of the planes concerned in our problem. The two roof surfaces have the same slope, and we shall assume that the tile faces are contained in the planes represented in Fig. 86 by $V_1T_1H_1$ and $V_2T_2H_2$. These planes cut each other in AB—the line of intersection. From the sketch it is obvious that the plan and elevation of the line of intersection would be at 90° to XY similar to Fig. 56 in the chapter on the "oblique line." Hence, in Fig. 87—which is the orthogonal representation of Fig. 86—*ab* is the plan and a^1b^1 the elevation of the line of intersection or "hip."

In order to place a cutting plane at 90° to this hip (in accordance with our previous explanation), we must have an auxiliary view—a side elevation—showing the *length and slope* of it.

This is constructed by rabatting the right-angled triangle

beneath AB into the ground plane. The vertex is then at B_0 and bB_0a is the rabatment.

When a "plane" cuts a "trace" at right angles, it does so in every view; therefore, if we first draw an edge view of the cutting plane at g^2j, at 90° to B_0a, then carry the *base* of this plane in plan through j at 90° to *ab* intersecting the H.T.'s at *e* and *f*, we have the elements of a triangle containing the angle between the planes.

To make clearer, draw the plan of g^2 on *ab* (where it lies), join *ge* and *gf*, then *egf* is the plan of the triangle contained by a right-angled section, *beneath* the two oblique planes. The

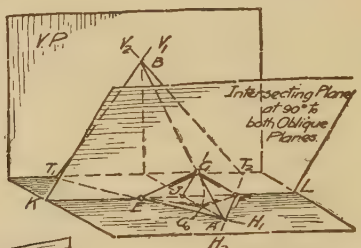


FIG 86

Sketch of Figs 86 & 87

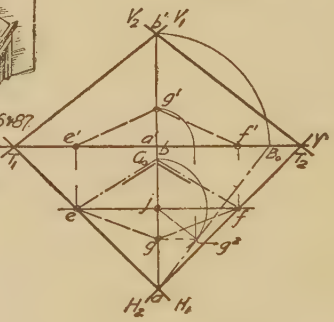


FIG 87

elevation of this is $e^1g^1f^1$ obtained by projection and measurement of the vertical height of g^1 above the plane.

Now observe that jg^2 is the perpendicular height of the triangle, which we have rabatted from its actual position into the ground at G_0. Then eG_0f is the angle between the planes.

Notice also, that the complete plan and elevation of the triangle EGF may be omitted from any such problem as it does not affect the result. *It does, however, assist you at first to grasp the position of the triangle whose true shape is your required solution.*

Fig. 86 should make the problem quite clear.

We may now assemble and abbreviate the steps in the process of solution:—

(*a*) Obtain a side view (rabatment) of the line of intersection of the planes.
(*b*) Make a right-angled edge cut from the line to H.P., so as to determine the height of the triangle of intersection.
(*c*) Draw the plan of the trace of this cutting plane along H.P. to intersect the given horizontal traces.
(*d*) Rabat the triangle, whose base and perpendicular height have been found, into the H.P.

Before giving examples we shall consider some further cases of intersecting planes of common practical occurrence.

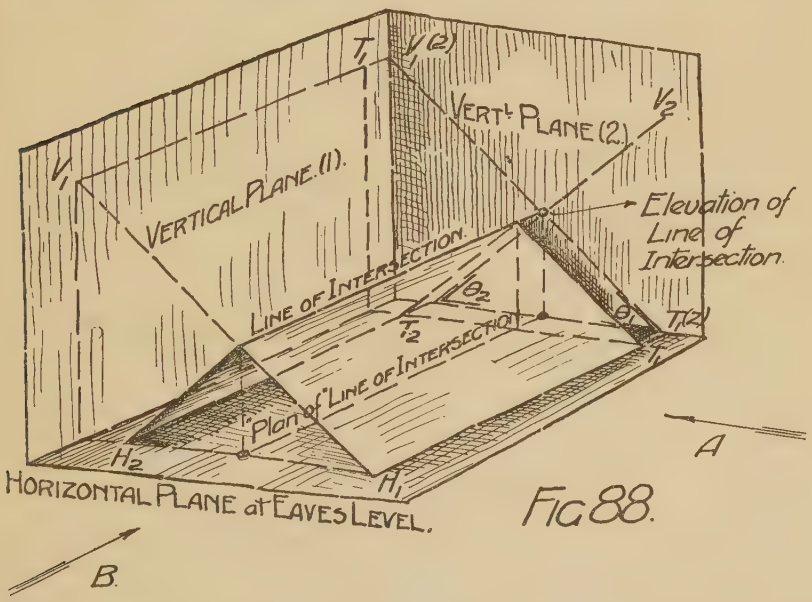
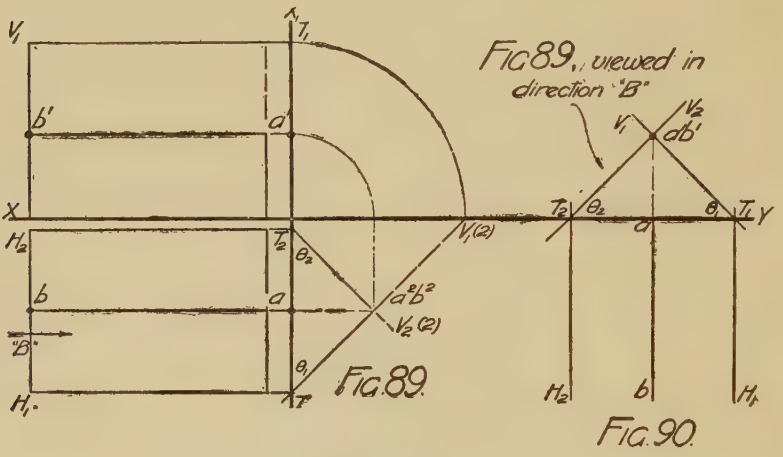


FIG 88.

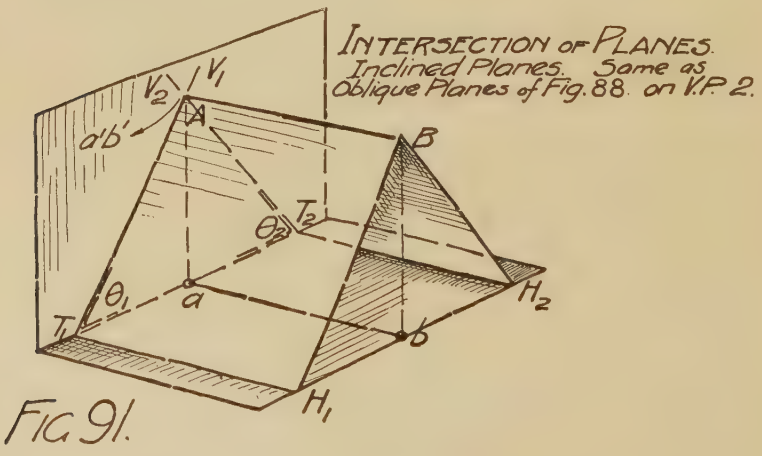
The commonest form of gable roof, with no break whatever in its surfaces, is illustrated in Fig. 88. As viewed from the front (*see* arrow A) the two roof planes are oblique planes, but of the simplest character, agreeing with Fig. 65 C. But if these same planes be viewed from the "end" (*see* arrow B), and their projection made on V.P. (2), they are simplified into two inclined planes, as explained in principle throughout chap. vi.

If we ignore V.P. (1) and make our projection on V.P. (2)

alone, showing the traces of the inclined planes, we obtain the condition adopted in practice with the view of reducing work to a minimum.



Figs. 88 and 89 illustrate the positions and slopes of these planes when considered as oblique planes, and Figs. 90 and 91 show the simplified planes.



You should *simplify wherever possible* in a similar manner, by consistently choosing the elevations easiest to conceive and manipulate.

In representing roof planes in work of a complicated

character the geometrical arrangement is of first importance, and needs careful study.

When you are treating a complete building, however, you cannot then choose your elevation of a roof to simplify your drawings, for the elevation of the roof must accompany the faces of the building and show their relation to each other.

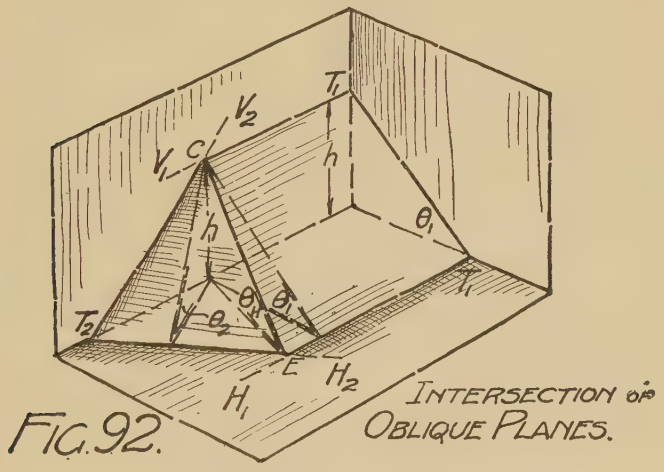


FIG. 92. INTERSECTION OF OBLIQUE PLANES.

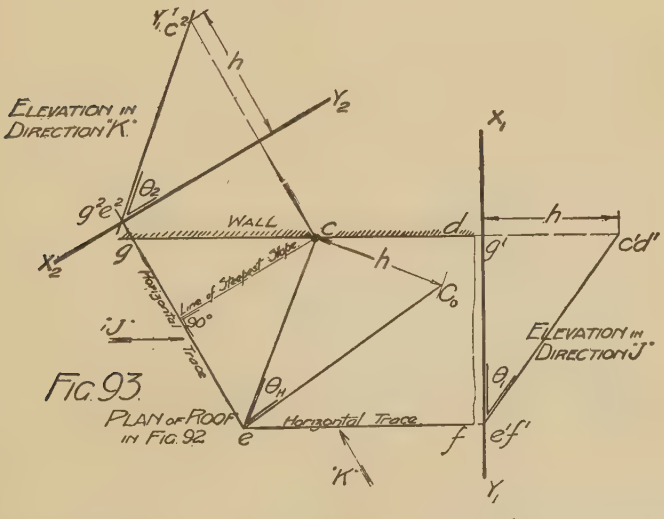


FIG. 93.

Fig. 92 illustrates the intersection of another pair of roof planes with the detailed working of the problem at Fig. 93. It is intended here that you should think of the plan of the

roof as being *given*, and also the rise of the roof above the eaves. This rise is denoted by *h* in the drawings.

If the roof plane, whose plan is *cdef*, is viewed in the direction of the arrow J, its "edge" is seen, and therefore its "true slope." This elevation is constructed on X_1Y_1, and the slope is shown by the angle θ_1.

We shall shortly deal with the angle between the two roof planes as a test example, but incidentally let us find the slope of the other roof plane (whose plan is *gce*), and the inclination of the hip rafter CE. Now we know the height of C, and that the eaves line (horizontal trace) is the same level at *ge* as at *ef*; therefore, by an auxiliary elevation in direction K we shall convert the roof plane from an "oblique" into an "inclined" plane and see its "edge view" and "true slope." Such an auxiliary elevation is shown on the ground line X_2Y_2 and the slope marked θ_2.

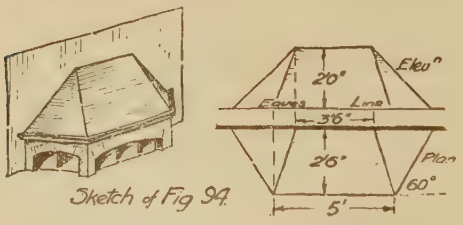
The hip slope is determined by solving the oblique line, and should be quite clear from previous work. Any difficulty in grasping the rabatted hip would probably be overcome by reference to Fig. 92.

EXAMPLES

(*Simpler Cases of Intersection*)

(1) Proceed with Fig. 93, and solve the angle between the two roof surfaces, which, according to the practical term, is called the "backing angle" for the hip rafter, or—when slightly corrected for slope—the angle for the hip tile; in geometry it is known as the "dihedral angle" between the roof planes.

(2) Fig. 94 shows a bay-window roof and is accompanied by a sketch.



Sketch of Fig 94

FIG 94
ROOF BAY WINDOW.

Its plan is fully given, and the ridge in elevation is 2 ft. above the eaves. Complete the two views as shown, then solve—

(*a*) The angle of inclination of both roof planes;
(*b*) The inclination of the hip;
(*c*) The dihedral angle between the hipped surfaces.

(3) Refer back to Fig. 24. Determine the angle between any pair of roof surfaces in this octagonal turret roof.

(4) Determine the angle between the adjacent pair of larger sloping surfaces in the stone gate-pier cap of Fig. 27.

(5) Fig. 34 illustrates a hipped roof. Find by construction the angle between the planes.

(6) Think of Fig. 5 as the timber roof of a spire. Determine the backing for one of the hip rafters, viz., the dihedral angle between an adjacent pair of roof surfaces.

(7) Fig. 8 is a hexagonal turret roof. Find—
(*a*) The hip backing;
(*b*) The length of an angle rafter.

(8) Fig. 9 illustrates a roof with two hipped ends. The right-hand end is roofed with a steeper slope than the other. Obtain the dihedral angle between a pair of roof surfaces at *each* end. Compare these angles. Which is the larger, and why?

(9) If a roof be pitched steeply and a second be flat pitched and both stand over the same plan, what difference does it make to the covering? Do you require *more* or less for the steep pitch? Why, then, are steeply-pitched roofs employed?

(10) Fig. 43 is a block of stone with different bevels at the sides and front. What is the "testing angle" between these two faces so as to apply like a try-square at 90° to the edge between the faces?

Valleys

In the earlier part of this chapter we have concentrated our attention and chosen our examples on the intersection of planes forming "external" roof angles or "hips." These angles are 'sharp" on the "exterior" of the building, because they *include less than* 180° *within the roof.* But roofs and other structural surfaces often meet each other so as to form an "internal" angle, including an angle of less than 180° on the "exterior" of the roof surfaces, *and therefore sharp "within" the roof.*

Such intersections are known as *valleys.* Some drawings containing valleys have been previously given, but we have only referred to the line of intersection and the shape of the surfaces as in Figs. 17, 35, 59, and 60.

We shall now consider the angle contained between the surfaces externally at a valley, in the same way as we have considered the hip. The use of this angle is to determine how much "bend" to give to the wings of the valley tile, so as to lay flat on the two surfaces of the roof adjacent to the valley.

(In detailed work, we need to consider the exact *slope of the tiles*, not the slope of the rafters.)

Fig. 95 is the typical "valley" formed by two intersecting lean-to roofs, or you may consider them as "parts of vee roofs."

Study this freehand sketch first. Recalling the first principle, remember that to find the angle between the planes

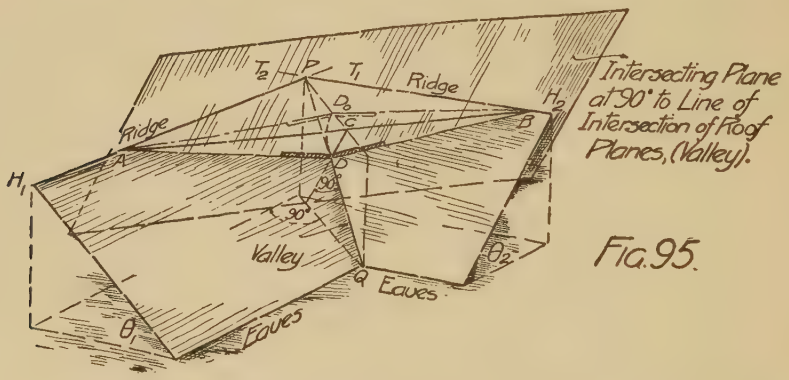


Fig. 95.

we must place a section plane at 90° to the line of intersection and *find the angle between the traces of this section plane on the roof planes.*

In the sketch the intersecting plane cuts the roof planes in the lines AD and DB, hence ADB is the angle between the surfaces.

The mode of attacking this problem, to be logical, should be the reverse of that for finding the hip angle. Think, therefore, of the horizontal plane as passing through and containing the *ridges*, not the eaves. Construct the edge view on this basis and measure the perpendicular distance from the line AB down to the valley at D, Fig. 95. The triangle ABD (whose base and perpendicular height from C are now known), must be rabatted upwards into the H.P. in the manner indicated behind the intersecting plane in the figure.

Fig. 96 contains the full geometrical working of the problem.

Draw the plan of the roofs. These may not have the same slopes, but in any case their line of intersection joins the intersection of the ridges and eaves. Given the plan of the eaves and ridges, and the height, or the angle of slope of one roof, the rest is determined by construction.

To solve the angle proceed as follows:—

Construct a rabatment of the valley whose plan is pq, at pQ_0, making Q_0 distant h from and at 90° to the plan because it is so much *lower* than P. Take any point on the rabatment

as D_0 and drop a perpendicular to meet the plan at c; then the trace of the cutting plane is at 90° to the plan through c,

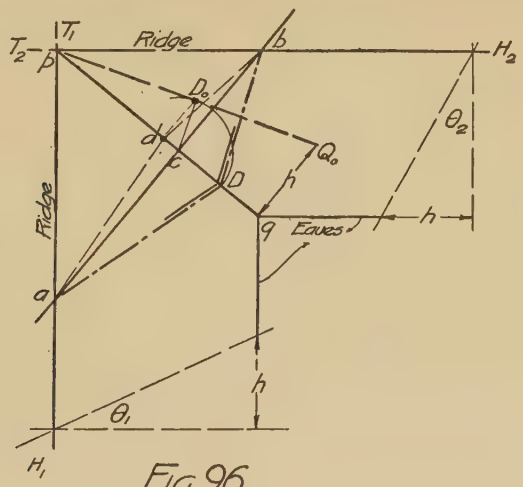
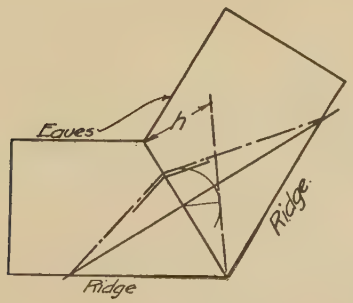


Fig. 96.

its plan being ab. Drop D_0 on to pq (and at 90° to it) at d. Join da and db. Then dab is the *plan* of the triangle whose angle at D is the inclination required. Rabat this triangle into

Fig. 97.



the H.P. by lifting it into the plane about its base, transferring cD_0 to the plan at cD. Then aDb is the angle between the two planes—the inclination of one plane to the other. (Observe that the triangle ADB may be rotated into the H.P. on *either* side of AB according to convenience. Both directions have been employed in Figs. 95 and 96 separately.)

A similar problem is worked at Fig. 97, where the angle

between the roofs in plan is not a right angle. Let the difference in height between ridge and eaves be h, as before. The same process has been followed, *omitting everything that is unnecessary to obtain the solution of the problem*. We may very briefly state the procedure, and leave the student to compare it with that for Fig. 96.

(*a*) Rabat the valley line.
(*b*) Place a perpendicular section in "edge view."
(*c*) Draw the base of this section at 90° to the plan of the valley to cut the ridges.
(*d*) Assemble the perpendicular height in edge view and the base line of the section in their correct positions when rabatted into H.P.

The vertical angle of the triangle is the dihedral angle between the planes on the outside of the roof.

EXAMPLES ON VALLEYS

(1) Find the dihedral angle at the valley in Fig. 17, chap. i. Full particulars of dimensions, etc., are given in the figure.

(2) Fig. 13 is a plan of a roof with three valleys. Solve the inclination between the roofs for one valley of each type.

(3) Roof with small gable at Fig. 10. What is the approximate angle between the wings of the valley tiles as obtained from the slope of the roofs adjoining the valley?

(4) Determine the angle between the main roof and the dormer roof illustrated in Fig. 35. Both roof pitches are 45°. How long is the valley in this example?

(5) Fig. 59. Two lean-to roofs for a conservatory, meeting at an acute angle in plan. Find the angle for determining the mitre between the two sashes, each 2 ins. thick, jointed at the valley line.

HINT.—*Obtain the dihedral angle, measure 2 ins. to represent thickness, and draw parallels to the angle obtained. For the mitre join the two angles between the faces of the sashes. Should the slopes of the adjoining roofs be the same, the mitre will bisect the angles.*

(6) Fig. 60 indicates a hip and valley roof. Required, the angle between the surfaces at the valley and the hip. Solve these separately and compare them. Is there any similarity between the "external angle of one" and the "internal angle of the other"?

CHAPTER VIII

INTERSECTIONS OF PLANES (*continued*)—REPRESENTATION OF PLANES BY "LINES OF SLOPE"

IT is unnecessary to always represent a plane by its two traces as we have previously done, for, when we know sufficient about sloping planes we soon come to notice that all of them are very similar, except in their position with relation to other planes—especially vertical ones.

Let us turn our attention to Fig. 98. This rough sketch shows a horizontal plane and an inclined plane rising from it. The horizontal trace is shown and marked H.T., but no vertical or other plane is introduced.

The slope of this plane to the H.P. is measured by a triangle standing vertically under the plane, *its plan being at* 90° *to H.T*

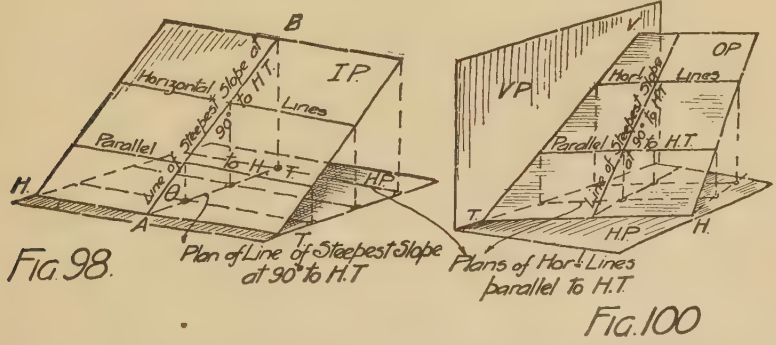


FIG. 98. FIG. 100

Previous work should have thoroughly established this fact. The hypotenuse of the triangle is the line AB in the figure. It is called the *line of steepest slope*, for if it be moved to the right or left about A and kept in the plane, the end B must become lower and the slope of AB correspondingly less.

Observe that *this line of steepest slope* is at 90° *to the H.T. of the inclined plane.* Also, as every oblique plane is simply an "inclined plane" *when the V.P. is removed*, we can fully represent any plane by the "horizontal trace" and "one line

of steepest slope," provided that the height of some point in the latter is shown or implied.

In general geometry, such a line of steepest slope is represented by its plan, using a double line evenly divided, and transforming it thereby into a scale of heights for known horizontal distances. This is not of special interest or immediate practical value to us, so that we may pass it by, but we shall use the principle and recognise the value of the shortened method of working, which allows of many lines being eliminated from the solution. This principle also allows of problems being set with a minimum of sketching and description which will be evident as we proceed.

Before leaving Fig. 98 notice that all lines contained in the inclined plane which are parallel to the H.T. are horizontal, and, conversely, all lines which are horizontal and contained in the plane are parallel to H.T. and to each other. An edge view of all such lines would be "points."

Fig. 99 represents a plane by its horizontal trace, line of steepest slope *ab*, and a knowledge of the height *h* of

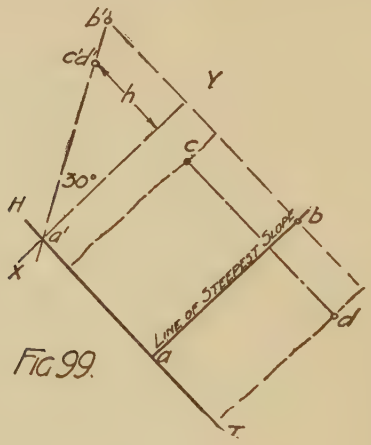


Fig 99.

some given point or line, as *cd*. If an edge elevation of the plane is required an XY is placed at 90° to H.T., and the auxiliary elevation projected upon it, c^1d^1 being measured to the height *h* and the edge of the plane drawn through it; a^1b^1 is the elevation of *ab*, and coincides with the edge of the plane, while c^1d^1 is the end elevation of the line *cd* in plan.

Fig. 100 shows an oblique plane with a "line of steepest slope" and a series of horizontal lines upon it. Study this figure, and sketch it with the vertical plane removed. You

will see that its removal in no way alters the value of the particulars supplied by the remaining plane and lines.

Turning our attention again to practical applications of the principle, we have in Fig. 101 two roof planes intersecting each

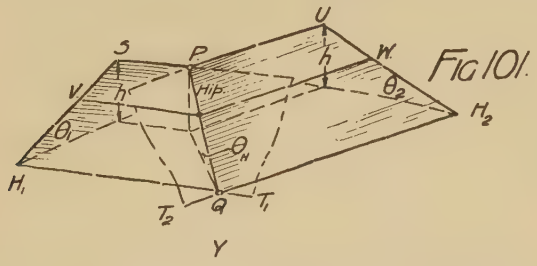


FIG 101.

other at a hip the planes being continued in dotted lines a short distance beyond the intersection at PQ.

In Fig. 102 the same roof planes are shown quite apart from any vertical plane. The horizontal traces are H_1T_1 and H_2T_2. They intersect at Q.

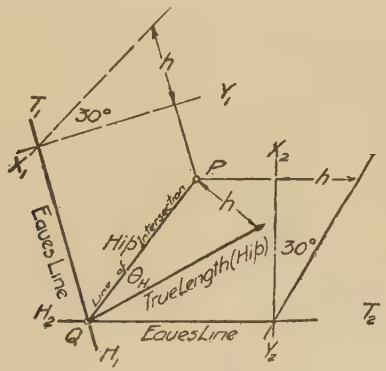


FIG 102.

The slope of the two roofs might be the same (as in Fig. 102), or, if different, θ_1 and θ_2 would define them, as in Fig. 101. Let the ridges be level and the height h known, then, whether the slopes be equal or not, a sectional elevation of each plane might be constructed, horizontal lines of the same height determined, *e.g.*, VW in Fig. 101, or again, SP, PU. Then the hip passes through their common intersection at the line PQ.

When two planes of the same slope intersect each other, they do so in a line, the plan of any point of which is equidistant from the horizontal line at the eaves. *Such a line is a*

bisector of the angle; therefore, if the planes are known to slope equally, you may at once draw the *hip* or *valley* to bisect the plan angle.

Work out a case like Fig. 102, taking any plan and with equal slopes. Set off these equal inclinations at the sectional elevations on X_1Y_1 and X_2Y_2. Take any point P at a position

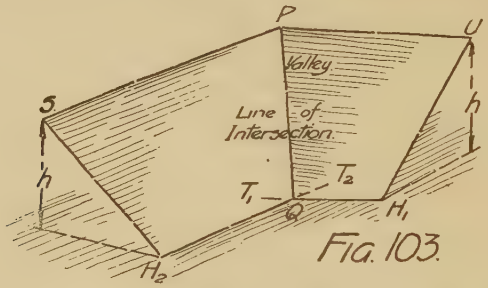


Fig. 103.

in plan where h units of height have been reached in each slope. Then P is found to be equidistant from the two eaves lines, and PQ is proved to bisect the angle.

Fig. 103 illustrates by a sketch two roofs of different inclinations forming a valley. A similar case (with a different plan angle probably) has been fully worked out in Fig. 104. The

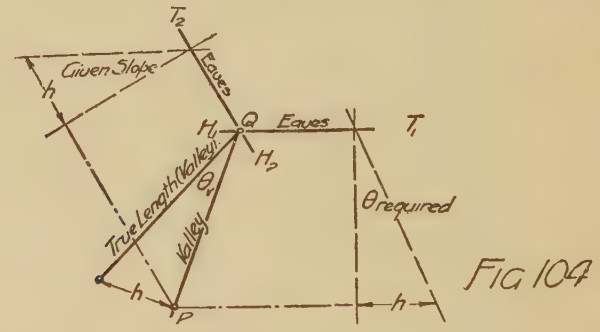


Fig 104

slope for the left-hand roof is given. Its "span" differs from the right-hand one, hence the "slopes" differ, because both have to reach the same height at the ridge. We are required to determine the slope of the right-hand roof and the length and inclination of the valley.

Let the stroke and dot lines represent the ridges, then the plan of the valley is obviously the line marked PQ. Construct a cross section of the right-hand roof making h to agree with the height measured at the ridge from the section of the "given roof." The base angle is the θ required.

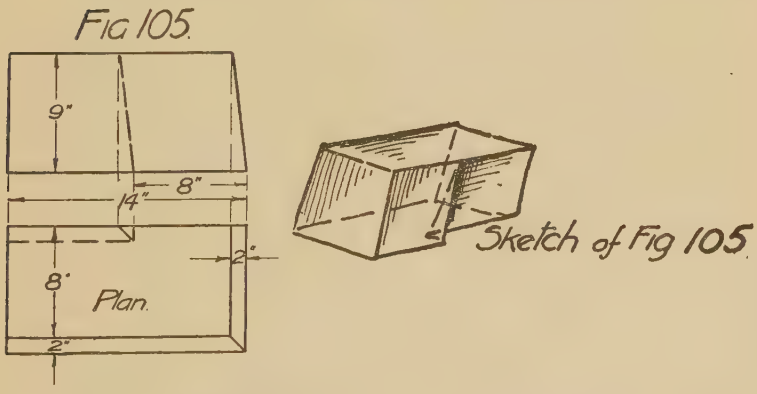
The length of the valley rafter and its inclination θ_v are determined as in previous problems.

We see, therefore, that the intersection of two roofs at a hip or valley may be determined and all the bevels obtained if we know the position of the "eaves," the slope of the surfaces in degrees, or the height of some horizontal line in the plan of each roof.

This principle will very often be met in later work, being applied to abbreviate the conditions of a problem and its solution to the least number of lines.

FURTHER EXAMPLES ON THE INTERSECTION OF PLANES

(1) Fig. 105 illustrates a quoin stone to a battering wall, such as the angle of a steeply-pitched spire. The next quoin block would be reversed in shape so that the quoins lapped on opposite sides to ensure a good bond. The "batter" or "slope" of the faces is the same inside and out, so that the thickness of the wall is constant. (Examine the



sketch attached to the figure to be certain of its form.) The two inner and outer faces intersect on the short diagonal lines of the plan. Find the true shape of the front face. Construct the angle between the outer faces.

(2) A bay-window is constructed at the splayed angle of a building (Fig. 106), the centre window being parallel to the splay, which makes 135° with the main wall internally. The plan of the bay is shown with the hipped roof indicated by dotted lines. Find the exact position of these lines when the slope of all three surfaces is 36°. Determine the angle between a pair of surfaces; name it and state its use. (*See* next problem.)

(3) In Fig. 106 assume that the ridge (between the dotted hips) is 2 ft. 6 ins. long, so that the exact positions of the "plans" of the

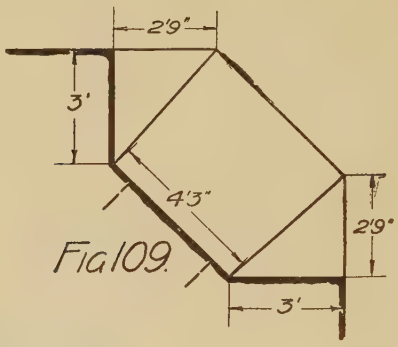


Fig 109.

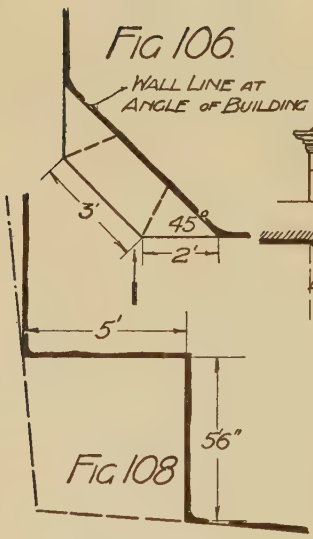


Fig 106.

Fig 108

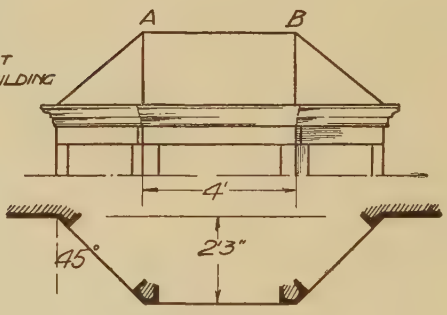
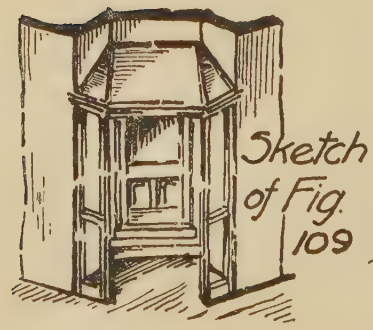


Fig 107.



Sketch of Fig. 109

Sketch (alternative) to Fig. 109.

hips are known. Let the centre surface be inclined at 30°. Find the pitch of the other surfaces and the angle between either pair.

(4) Fig. 107 shows the plan and elevation of a bay-window roof, so arranged that the elevations of the hips are vertical lines. Let AB be 2 ft. above the eaves. Draw the plans and find the inclinations of the "return roofs." Construct the angle between the return and front surfaces.

(5) The plan of a square-recessed angle to a building of irregular plan is shown at Fig. 108. The building line is indicated by dotted lines enclosing an irregular figure. It is decided to erect a porch over this angle finished with a roof having two slopes. It may be done in two ways. Think out the forms and sketch them in free-hand isometric. Then draw the plan of each and solve all the bevels you can remember to have required in previous problems. Name each result.

HINT.—*Examine the two sketches attached to Fig.* 109. *These should help your ideas. Your results should give* (1) *a hip roof;* (2) *a valley roof. Which gives the best appearance?*

(6) Fig. 109 illustrates a somewhat similar though more difficult case than the last. It is a proposal for the formation of an open porch at a recessed angle. Fig. 109 (diagram) is the plan of building with the "wall face" lined in black and the doorway indicated by dotted lines. The porch roof is outlined for eaves and mitres, and these latter may represent either hips or valleys according to the conception of the method of roofing. Two sketches are supplied, showing alternative designs for the roof. Work through each case, and obtain the dihedral angles at the hip and valley. Compare your results.

CHAPTER IX

DEVELOPMENT OF STRAIGHT LINE SOLIDS

SOLIDS bounded by plane surfaces which intersect each other in straight lines are sometimes described as "straight line solids."

Before reaching this stage you have become acquainted with the commoner forms such as right prisms and pyramids, and most probably have laid out on one plane the surface bounding the solid form.

If not, you should take models and roll them over on a plane surface "without sliding," and mark the shape of every face as it comes in contact with the surface. The result is a "development."

We must now consider these a little further to form a basis on which we may proceed to the development of roof planes and of curved surfaces. Our desire is, not merely to develop the plane surfaces of these solids, but to indicate further the change of form in the developed surface when the solid is severed by a cutting plane.

Irregular boundaries, so formed, have to be geometrically solved or otherwise considered in constructional work.

SECTIONAL PRISMS

As an introduction to the subject consider the irregular prism illustrated in Fig. 110. It is a combination of forms such as occur in roofing.

The solid is intersected by a vertical plane shown at SN in plan. To realise the form of the cut surface at this intersection refer to the sketch above the figure. Let this solid be rolled (without slipping) from left to right, as shown by the arrow in elevation. We desire to lay out on the flat the shape of the plane surfaces of the solid (except the base) in one piece, and to show the "line of cut" made by SN across the faces.

Taking the order in which the surfaces come in contact with the horizontal plane, set out the lengths of the edges ED, DC, CB, and BA, along the horizontal line at *ae* produced. Being a "prism" the surfaces are all bounded by pairs of

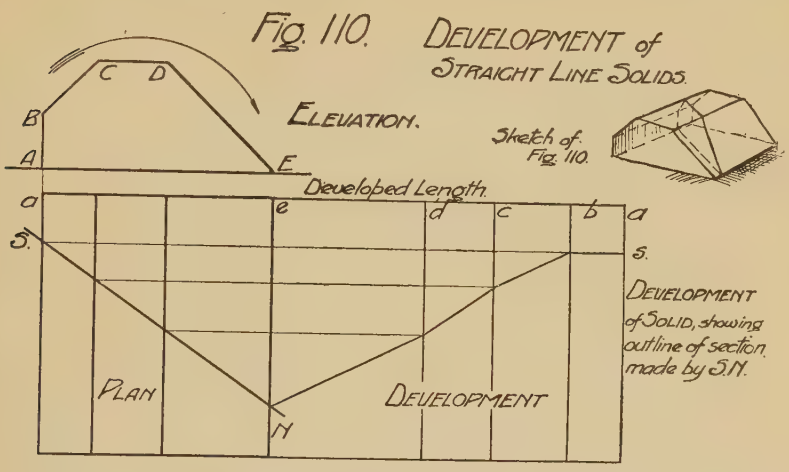
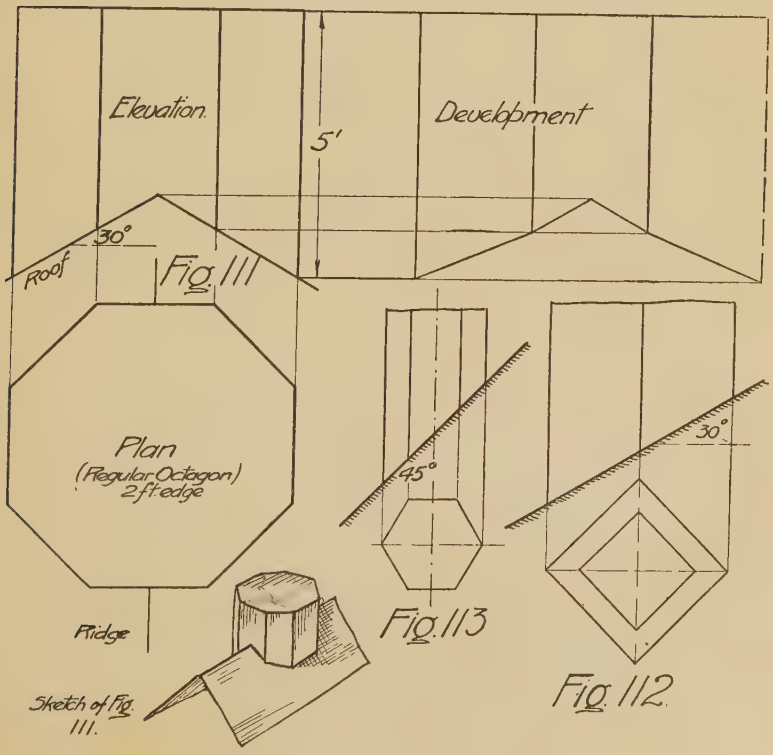


Fig. 110. DEVELOPMENT of STRAIGHT LINE SOLIDS.
C D
B
A
E
ELEVATION.
Sketch of Fig. 110.
Developed Length
a
e
d
c
b
a
S.
S.
DEVELOPMENT of SOLID, showing outline of section made by S.N.
PLAN
DEVELOPMENT
N


Elevation.
5'
Development
30°
Roof
Fig. 111
Plan
(Regular Octagon)
2 ft edge
Ridge
Sketch of Fig. 111.
45°
Fig. 113
30°
Fig. 112.

parallel edges; hence the series of parallel lines to the right of the plan, which, with the "projected length," cuts off the rectangle representing the total area.

As the plan determines the distance from the end at which the section plane cuts each "edge," a horizontal projector from the intersections in the plan to the corresponding points on the "development" fixes the points through which the straight lines of intersection pass.

If you have still difficulty in following the procedure draw the development, cut out the paper, and fold it over your plan.

After studying Fig. 110, *and the sketch carefully, construct a definition of "development."*

Many constructional examples occur where regular solids are intersected by planes and require development of their surfaces to be accomplished. The practical necessity for this operation depends upon the nature of the structure. Should the form need to be constructed in wood or metal a geometrical development may be indispensable, but if the shapes are required for "mensuration" only, the dimensions from which the surface area may be calculated are usually clear in the plan or elevation, or both.

Fig. 111 is a typical example. It shows an octagonal turret base rising through a vee roof centrally at the ridge. The conditions are shown in single lines only. To make the development on paper, lay out the surfaces on a vertical plane taking them in any convenient order, and building upon one of the vertical edges in elevation. One half of the base has been developed. Study it, and then proceed to the following examples.

EXAMPLES

(1) Turn back to Fig. 38. Develop the whole of the surface of the right-hand piece of the octagonal prism below the section plane, EF.

(2) Fig. 41 is similar to Fig. 111, but more detailed. As an exercise determine the developed shape of the "exposed surface" of the ventilator base when the height above the ridge is 15 ins., and the ridge rises $1\frac{1}{2}$ ins. above the rafters. Add $\frac{3}{8}$-in. roof boarding to the figure, and take it into account.

(3) A square chimney, or other shaft, is placed diagonally with regard to the eaves and roof spars, Fig. 112. Let its faces be 2 ft. 3 ins. wide. Determine the shape of the developed vertical surfaces.

(4) In Fig. 113 is illustrated a hexagonal shaft rising through an inclined roof plane. Lay out the shape of the surfaces of the shaft which are above the roof. Sides of shaft are 1 ft. 2 ins. wide.

(5) Set out Fig. 110 to any dimensions you please. Take the section plane SN reversed so as to slope the other way, and place it at 30° to the square end, passing through *e*. Develop the surface.

Sectional Pyramids

Suppose the development of the sloping surfaces of a pyramid to be required. Lay the pyramid on one of its triangular faces as indicated in the sketch, Fig. 114. Mark its outline and turn it over from face to face, marking each successively on the plane. The result is the dotted outline on the ground plane in front of the solid.

Such development—either whole or partial—is often required for practical use, and we need some method of construction applicable to drawing paper, or to the setting out floor as required.

Take a plan and elevation as Fig. 114. The particulars of the pyramid supplied by this drawing are: shape of plan, (square), the vertical height of the pyramid, and the slope of its sides. The given views do not show the true shape of any surface except the base, with which we are only indirectly concerned. We have first to find a method of obtaining the true shape of the triangular faces.

> *Method* 1.—By an auxiliary plan (Fig. 114). If one of the triangular faces be viewed at 90° to its plane, its true shape must be seen; hence, project an auxiliary plan on a plane passing through v^1e^1 (really a rabatment) by projecting at 90° to v^1e^1 and measuring on these projectors the distance of the plans *ve* in front of XY, and complete the figure $V_DB_DD_D$. Its centre line VE (at 90° to the base from the vertex) is the perpendicular height of the triangle.

You should now see how easily this triangle could be constructed by using the known base and position of E, and the slant height of the triangular face. These may be transferred and constructed "anywhere."

Being a "square" pyramid, the addition of three similar triangles to the edge of the first one completes the development of the slant surfaces. Notice how this is done. All the angles are of the same length—because the pyramid is a "right pyramid," its vertex being perpendicularly over the centre of the base—therefore all the angles $D_DC_DA_DB_D$ are on the circumference of a circle whose radius is V_DB_D (*the real length of the angle*). The chords B_DD_D are equal in length to the base edges.

> *Method* 2.—By auxiliary elevation or rabatment of an edge in plan, find, by rabatment, the length

of one of the edges as vD in plan. On this build the triangle by striking the curve and cutting off chordal lengths as before. (No figure.)

Method 3.—Draw the plan and elevation so that the diagonals in plan are "one parallel," and the other at 90° to the V.P. The elevation then shows the true length of an angle, and we can at once use this to build the development upon, knowing the base edges, etc., as before. Fig. 115 shows this method.

NOTE.—To make the best use of your geometry you must be able to take advantage of any method which offers the quickest way to a solution. The "choice of method" rests with yourself, and must depend on the conditions of the problem, the means at hand for its solution, and for what purpose the original drawings may have been arranged.

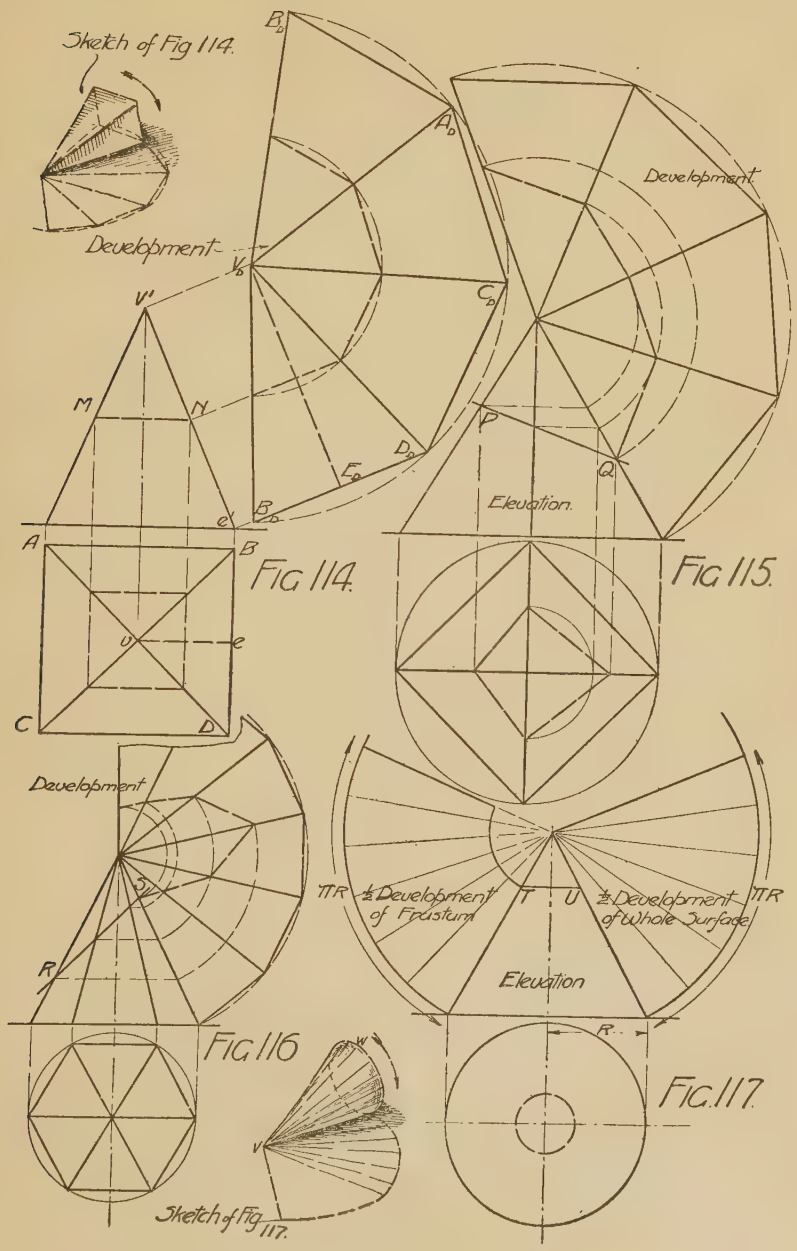
Referring again to Fig. 114, let the pyramid be severed by a plane parallel to its base, as, for instance, MN. What is the shape of the surface between the planes MN and H.P.? *This part of a pyramid is called a "frustum"*; its top face is parallel to the base. The portion removed is a small pyramid and part of the larger one, hence its surfaces form part of the larger surfaces of the original pyramid. Determine, therefore, the position of the horizontal edge through N, place it on the auxiliary plan between $V_D B_D$, and proceed to repeat it by transferring with the compasses round the remaining sides. The strong dotted line shows the resulting boundary to the frustum.

In passing we should observe that the area of any part of the surface of these solids may be obtained from the development by plane mensuration.

So far every development we have constructed, and every sectional part of it, has been capable of mensuration by the "triangle rule" or the "trapezoid rule." These you are acquainted with from vol. i., and a few examples are appended for revision and further application.

Instead of the point of the pyramid having been removed by a horizontal plane, as in Fig. 114, the pyramid might have penetrated an inclined plane resulting in the section PQ, Fig. 115. We have previously studied sections made by such planes, and we have now only to find the outline of the upper pyramid or of the remainder (which is referred to as a "truncated pyramid").

Because the outside inclined lines in elevation are of true length, we can (by the compasses) transfer these lengths to the correct angles in the development, but the cut across


Sketch of Fig 114.
Development
Development
Elevation.
Fig 114.
Fig 115.
Development
Elevation
Fig 116
Fig 117.
Sketch of Fig 117.

the centre line (representing back and front edges) causes a difficulty, for we cannot determine its exact position on the angle in plan by a vertical projection as in the case of the external lines, nor do we know the length from the vertex to the point of intersection; but, as all the angles are of equal length, and the "height of the points of intersection are known," a horizontal projector from the centre point to the outer angle determines the real distance from the vertex. This is transferred to the corresponding developed angles by the compasses, and the broken line of section may now be drawn. References are purposely omitted in Fig. 115 with the intention of the student thinking more for himself.

Should the pyramid be rectangular or irregular at the base, and a diagonal position of the angles in plan causes similar vertical lines to occur to those of Fig. 115, an auxiliary (side) elevation will always determine the position by projection and measurement of heights. The auxiliary view should, of course, be on a plane parallel to the line(s) whose intersection with the plane is (are) being considered.

The principle is further illustrated in the development of a regular hexagonal pyramid, Fig. 116, where the solid is cut by an inclined plane RS intercepting a portion near the vertex. The full development of the slant surface is given, and also the broken line due to the intersection. Again note, that because the angles are all of equal length, the two intermediate points are projected horizontally to the angle containing S, thus determining their true distances from the vertex at the point of intersection. As before, these are transferred by arcs of circles to their respective angles. The development is "partial," to economise space.

The full treatment of the development of the cone must be left to a later chapter, but, because of the similarity in the development of the cone and pyramid, we make a brief reference here.

A right cone is *practically* a "right pyramid whose base has an infinite number of edges." If you can imagine a pyramid having the number of triangular faces indefinitely increased you will see that it eventually becomes a cone.

Let the cone be laid on a generator, as in the sketch of Fig. 117, and rolled like the pyramid, without slipping, and it is at once clear how the development may be made. It consists of a "sector" having a radius equal to the length of the "generator" or slant height of the cone; and the arc of the sector equals the circumference of the base of the cone. This is illustrated to the right of Fig. 117.

If a frustum be formed by the horizontal plane TU

removing the vertex, its development is obviously that of the large cone, minus the surface of the smaller one. The arc of revolution of the small cone is necessarily parallel to the base arc of the large cone. See the half development to the left of the figure. For further consideration of the cone see chap. xii.

EXAMPLES

(1) A roof to a tower is in the form of a square pyramid frustum, as per Fig. 117*a*. Suppose you were about to make a model of this feature, lay out in one figure the shape of the four roof surfaces.

HINT.—Complete the pyramid and proceed as in Fig. 114.

(2) An octagonal tower has a base diameter of 6 ft. between its parallel faces, and rises to a height of 15 ft. It is so arranged that the pyramidal roof has its eaves 3 ft. above the main roof, and the back surfaces of the pyramid intersect the roof. The support at the eaves is shown in side elevation at Fig. 117*b*. Draw the front elevation and develop the pyramid faces to a small scale.

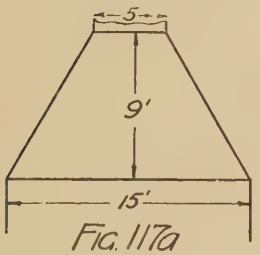


Fig. 117a

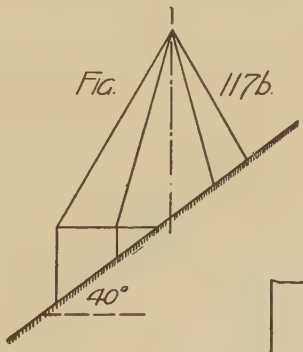


Fig. 117b

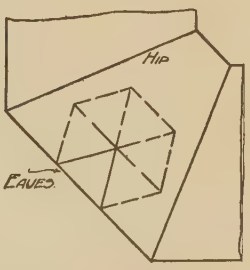


Fig. 117c

(3) The plan of a right pyramid is a rectangle $4' \times 6'$. Its vertical height is 10 ft. If cut by a plane 6 ft. above the base, develop the inclined faces of the frustum. State their total area in square feet.

Answer, 86.4 sq. ft.

(4) Plan at the angle of a building, Fig. 117*c*. Determine the development of the hexagonal pyramid roof which springs from the same eaves as the main roof. The roof pitch is 30°; base of hexagon, 3 ft. edge;

total rise of pyramid, 12 ft. A flagstaff 8 ins. diameter across the hexagon (at the diagonal) is to be allowed for, and the surfaces which rise above the roof developed.

(5) Calculate the surface area of the square pyramid of Fig. 114, assuming its base to be 5 ft. square, and the perpendicular height 8 ft. Sloping surfaces only to be calculated. *Answer*, 83.8 sq. ft.

(6) Fig. 117*a*, Example 1, is to be covered with slating. Find the nett area covered, stating your result in square yards.
Answer, $45\frac{7}{9}$ sq. yds.

(7) Work the example of Fig. 116, developing the complete surface when the edge of base is 6 ft., and the vertical height 12 ft. Measure your area from the laid-out surfaces. Then check by calculation, utilising the information gained in vol. i. *Answer*, 235.4 sq. ft.

CHAPTER X

FIGURES IN INCLINED PLANES

In the last chapter we have studied the development of a few straight line solids (complete and sectional) showing "one" method of finding the assembled form of several surfaces intersecting each other, and thus forming the bounding surfaces of a solid.

Think again of the slanting surfaces of a prism or a pyramid. We might have found the true shape of these surfaces separately and kept them isolated. Each would have represented the development of a single surface, though we more often use the term for an assemblage of surfaces which, by folding in some preconceived way, will produce the external form of a solid.

We shall now consider "isolated development," viz., the "laying out" of such surfaces and any arrangement of secondary surfaces contained therein. The plane faces we are called upon to develop will usually be rabatted about some convenient line (or edge) into the horizontal or vertical plane.

In constructive problems we are often in possession of two projections of figures which do not show their true shape—as you have already proved by the large number of such cases given in foregoing chapters—and in splayed work particularly we meet with them.

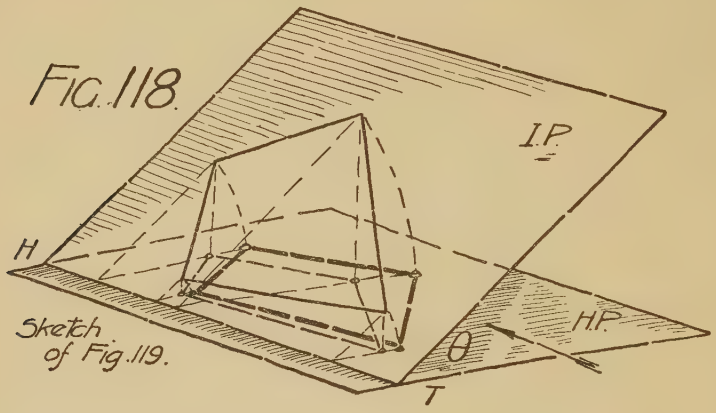
Observe—as proved by previous work—that an edge view of the surface to be developed is usually required whatever type of solution we employ. The principle of "rotation" or "rabatment" continues to require this as a first condition.

For example, study Fig. 118, which is a sketch of a H.P. and an inclined plane intersecting in H.T. The inclined plane contains an irregular quadrilateral figure, and we desire to obtain its true shape.

If we know the slope of the plane containing the figure we can construct an edge view of it, and we know that the edge view of figure and plane coincide.

Taking the conditions as represented in Fig. 118, we may obtain the true shape of the figure by rotating the plane containing it about its horizontal trace (or any other horizontal

line in it) into the H.P. The sketch we are considering shows "how this rotation takes place," the "final position of the revolved figure in the ground plane," and the "circular path



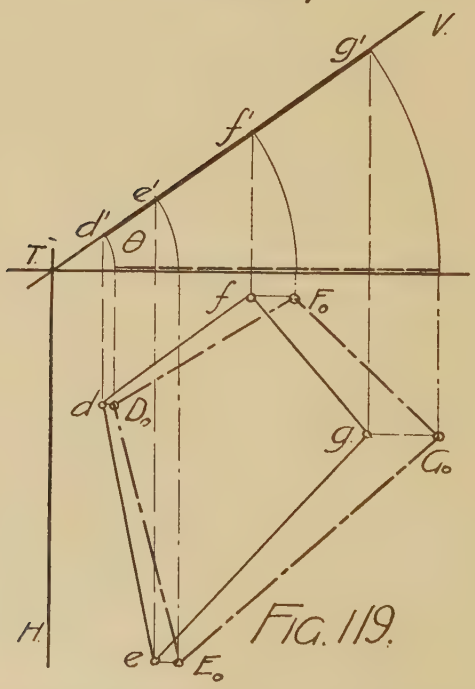


FIG. 119.

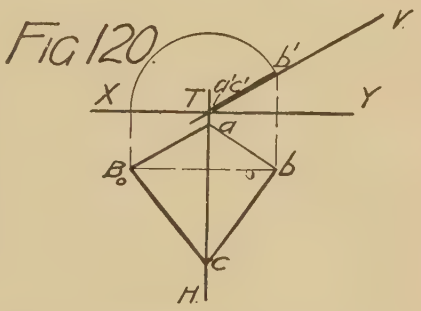
through which each point travels in reaching the ground." The "plan" (immediately below the figure) and the "developed outline" are both shown below the inclined plane.

Now connect these with Fig. 119. The given plan is *defg*; the inclined plane, slope θ, is VTH, and the elevation of the figure on V.T. is $d^1e^1f^1g^1$.

Observe that the circular movement of every point is truly visible when the direction of view is parallel to the H.T. (*see* arrow, Fig. 118), hence points $d^1e^1f^1g^1$ are rotated by describing arcs having the H.T. (hinge line) as their common centre, and whose "end" in the elevation is point T.

The distance of each rotated point from the H.T. is, therefore, established; then, the projectors parallel to H.T. from the foot of each arc are the lines on which the developed points lie.

When viewed from above, the arcs are in vertical planes immediately under the lines of steepest slope, and the points appear to travel outwards from their original positions at 90° to the horizontal trace; hence, the intersection of corresponding projectors from "plan" and "rotated elevation" determines the rabatment of the angular points. $D_0E_0F_0G_0$ is the complete rabatted figure.



If thought advisable to keep the "plan" and the "rabatment" quite distinct from each other, the inclined plane might be rotated in the opposite direction until it lies on the H.P. at the other side of the horizontal trace.

Fig. 120 shows this operation. The plan and edge elevation of a triangle are given, and the true shape is desired; its containing plane is revolved to the left until the triangle is on the ground at aB_0c. This arrangement prevents overlapping of the views, but requires more space than the former arrangement.

It is now desirable and convenient to apply this principle to the determination of further bevels in roofing, such as are dependent on the solution of the true shape of the "whole" or a "part" of the roof plane. (Some of these problems may be solved equally well by previously explained methods when using drawing paper and working to a small scale.)

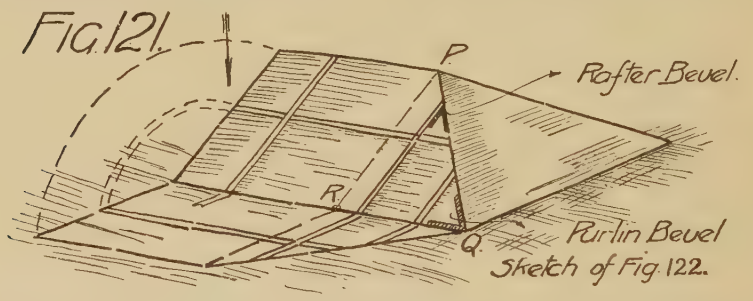
Consider the usual method of constructing a hipped or "any" broken roof.

The general method of construction is to provide a system of light parallel timbers (common rafters) following the "true slope" of the roof, so laid that the plans of these rafters are at 90° to the eaves line. They occupy the position of lines of steepest slope.

Fig. 121 represents a common hipped roof, and shows three of these rafters, two of which fit against the side of the hip, and are named "jack rafters."

The common rafters, where necessary, are supported at intermediate points by horizontal timbers (purlins), which receive *their* support from walls, trusses, or hip rafters, and may be set vertically or at 90° to the slope. In trussed roofs the latter is the more common and practical arrangement. One such horizontal member is shown in the sketch, its near end fitting to the side of the hip and crossing over the line PR which may be thought to represent a "principal rafter of the truss."

Carefully study Fig. 121. The true shape of the roof surface, with the rafters and purlins drawn upon it, must show the "fitting angle" of the member for marking and cutting. Thus the angle between any "rafter edge" and the "hip" is



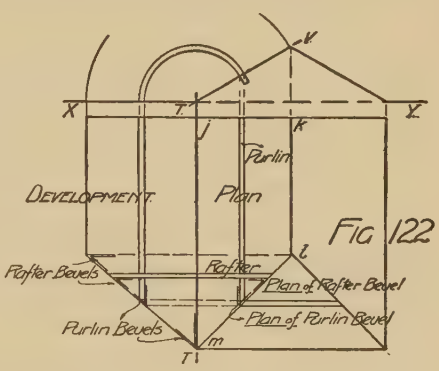
the "jack rafter bevel" to be marked on the "top edge." (From vol. i. p. 111 you know that the vertical cut of any rafter to fit the hip or ridge is the "angle between the roof slope and a vertical line.")

Again, the angle between the "purlin edge" and the "hip line" is the "top edge purlin bevel"; therefore, if we construct the true shape of the whole of the roof surface by rotating it into the H.P. (as illustrated in the sketch of Fig. 121) these main angles or "bevels" are at once truly visible. Fig. 122 shows the surface development correctly constructed by projection. The plan and end elevation are first drawn, assuming that the slopes are uniform and the hips bisect the angles between the wall lines in plan. Insert the purlin in the end elevation, obtain its plan and carry it across the return end. Notice that the eaves line is the horizontal trace of the plane to be developed. Rotate it in edge elevation taking the purlin width along with the plane. Remembering what occurred in Figs. 118 and 119 you will now realise that the plan points appear to move at 90° to the eaves until they reach the

distance outwards determined by the projectors from the rotated edge of the plane.

Noting which are the *plans* of the angles sought, their corresponding angles in the development are fairly obvious.

You should also realise that we have alternatives to the exact detail explained in Fig. 122. For instance, the figure could be revolved about "any" horizontal line that may be convenient, *e.g.*, the "ridge" or the "edge of a purlin." Where the purlin bevel is required for full size setting out, a small portion of the plan only is needed at the joint with the hip, and the inclination of its faces need to be known; then the development becomes a simple exercise, one case only of which is illustrated at Fig. 156, vol. i., being the practical and advisable abbreviation of the main general principle.



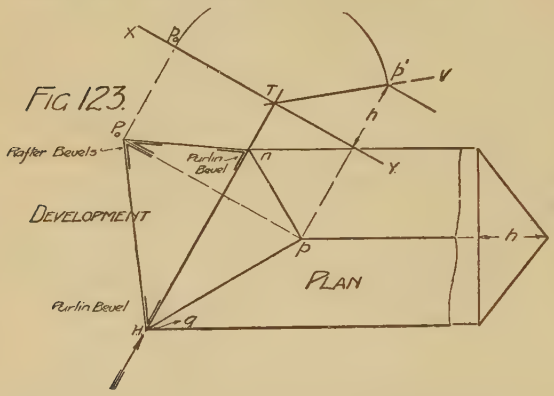
The jack rafter bevel is also capable of shortened labour, and this was also illustrated at Fig. 155, vol. i. The abbreviation consists in developing the "*figure in the plane*," *instead of the whole surface.* Both cases are continually applied in roof construction.

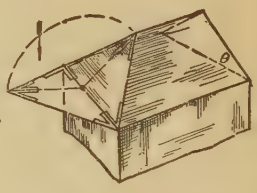
The point of importance here is that you should see that all these "methods" are variations of *one general principle*, and if you are dealing with a complete roof and its surface developments you should be able to point out at once which angles on the full surface are the required bevels for any member lying in, or parallel to, the surface.

Fig. 123 deals with the development of the roof over a plan with a "skew" end; a sketch, Fig. 124, further explains its "form."

Suppose the plan and cross section of a roof to be given as in Fig. 123 (to the right). The section supplies (or determines) the height of the ridge *h*, and we know that the height of p in plan is *h* units. Before we can develop any surface we need an edge view of it, hence an auxiliary elevation must first be constructed viewed parallel to H.T. Produce the eaves as H.T., draw XY at 90° to it, project from p and measure the height *h* above XY at p^1. Then p^1T is the edge view of the "end roof surface." Rotate Tp^1 on to XY; then perpendiculars drawn respectively from p_0 to XY and p to

H.T. determine the position of P_0 and P_0qn is the true shape of the hipped surface. Notice that the bevels for purlins and rafters on each side of the developed surface differ very much



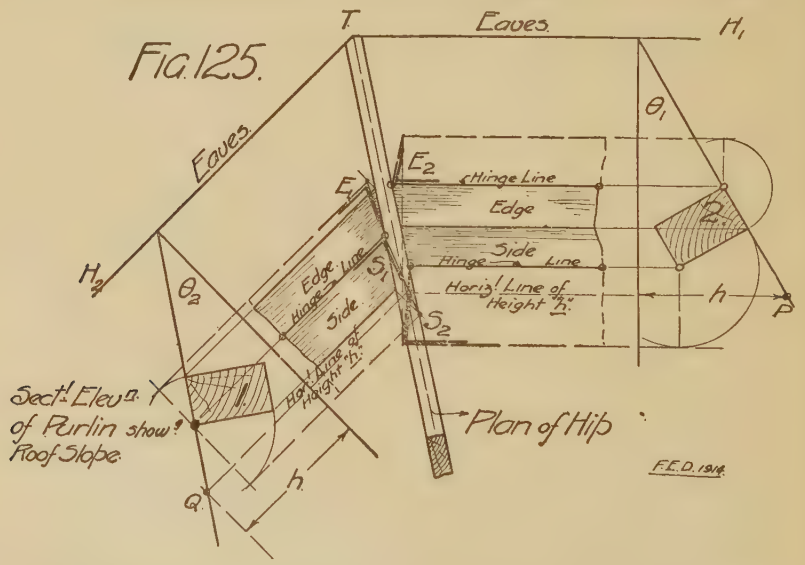


Sketch of Fig. 123.

Fig. 124.

owing to the lengths of the hips varying. (*See* example 2 in the following series for further reference to Fig. 123.)

In order to illustrate the solution of purlin bevels in detail for a hip *whose plan does not bisect the plan angle between the eaves*



a further case is given at Fig. 125. Eaves and hip are shown, and two cross sections of the roof give the inclinations θ_1 and θ_2.

Taking purlins of a known size, say $4'' \times 9''$, find the bevels

to mark on the timbers so as to ensure fitting against the hip while maintaining the top edges in the roof planes. Before obtaining these bevels we have shown incidentally how to find the plan of the hip when the varying slopes of the two roofs are given. Thus, set off the angles of inclination θ_1 and θ_2 at the two cross sections. Take any point P, height h units above the horizontal; let this be the end elevation of a horizontal line in the plane, and draw its plan parallel to the eaves. Find a point Q in the other slope having the same height h, by setting up h and projecting horizontally to meet the inclination. Let this be the end of a horizontal line in the other roof; it has the same height as the first. Draw its plan. The intersection of the two plans is a "point (not lettered) in the plan of the hip." Half the width of the latter may now be placed on each side.

Place the cross section of a purlin on each slope as shown. (They may conveniently have their middle edges intersecting on the centre of the hip.) Project the plan of each purlin. Then rotate each of the upper surfaces into the H.P. On the "left" the rotations are made about the middle edge; on the "right" about the outside edges of the purlin to avoid unnecessary crossing of the lines. The latter method is perhaps the least confusing.

E_1 is the edge bevel and S_1 the side bevel of the purlin numbered 1; E_2 and S_2 similar bevels for the purlin numbered 2.

EXAMPLES OF ROOF DEVELOPMENT AND MISCELLANEOUS PROBLEMS BEARING ON PREVIOUS WORK

(1) Fig. 126 is the plan and part section of a hipped roof to a square end, the "return" end having a different slope to the "main roof." Determine the development of "one side" and the "return," and mark the angles for cutting jack rafters and purlins.

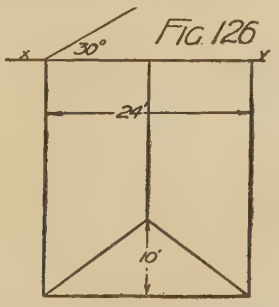


FIG. 126

(2) Develop front and back surfaces of the roof of Fig. 123, after working through the figure as given and described in the text. Mark all useful angles and *state* their use.

(3) Fig. 127. The roof to a porch is arranged as per diagram with one hip. The height of the ridge AB is 2 ft. 6 ins. above the eaves. Obtain—

(*a*) Length and slope of hip. *Answer*, 5.6 ft. and 26½°.
(*b*) True shape of each roof surface.
(*c*) Glazing bar bevels at the hip stiles.
(*d*) Area of two surfaces. *Answer*, 29 sq. ft.

(4) Given the plan of a bay window roof, Fig. 128. The positions of the hips are only "suggested" ones. If the centre surface is inclined at 30°, and the other two at 40°, find their position and develop the surfaces (without repetition of work). Mark the bevels and state their uses.

(5) Fig. 129 represents a right-angled triangle, which is contained in a vertical plane, VTH. Determine its true shape by rotation into V.P., as suggested in plan by the two short arcs.

(6) Fig. 130 shows the elevation and vertical section of a louvre ventilator to be fitted into the gable of a north-light roof, as illustrated by the attached sketch. The louvres are pitched at 45°, and the "bottom edge of one louvre" overlaps "the top edge of the next" by 1 in. The bottom louvre overlaps the frame by a similar amount. Determine—

(*a*) The correct setting out of the "section."
(*b*) The completed elevation to the given dimensions.
(*c*) The true shape of the louvre A by rotating it about the front edge into a V.P.
(*d*) The true shape of the front edge of the louvre.

Compare the angle of bevel in *b* and *c*, and explain why, in this case, they are alike.

(7) A window has an opening 4′ × 2′ 6″ wide. Its side linings are ⅞ in. thick, and arranged as shown in horizontal section at Fig. 131. Point A is 3 ins. to the right of B in elevation. To any good scale draw the elevation of the window, giving the top lining a bevel identical with the sides. Obtain the visible parts of the top and side linings.

(8) In Question 7 let the head lining be splayed 1 in. in 10 ins. Determine the shapes of the front and side linings.

(9) Fig. 132 is the plan and elevation of a hipped lean-to roof with a splayed end. Develop the roof surfaces by rotation into the H.P. Find the hip and jack rafter bevels completely.

(10) The plan and elevation of a rectangular hopper is shown at Fig. 133. Find by construction—

(*a*) True shape of outside face marked A. (You will need a side elevation first.) What use is this shape?
(*b*) True angle of mitre to be applied to the top edge B.
(*c*) The angle between the pairs of surfaces. Is the latter of any practical value?
(*d*) The internal capacity of the hopper. *Answer*, 16.6 cu. ft.

(11) Arrange a triangular ventilator in elevation and section, the latter to be exactly similar in section to Fig. 130, and the elevation to be

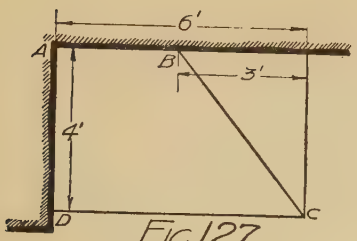
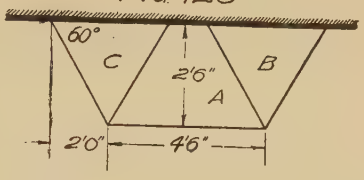


FIG 127

FIG 128



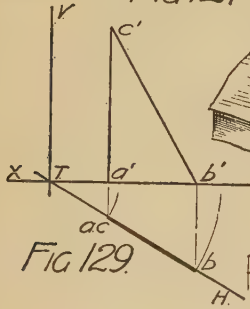


FIG 129

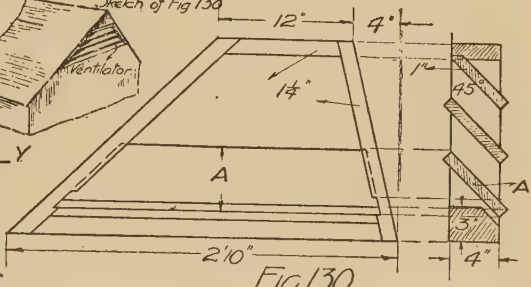


FIG 130

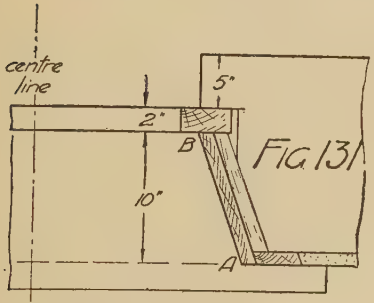


FIG 131

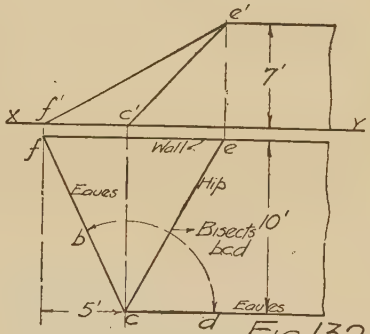


FIG 132

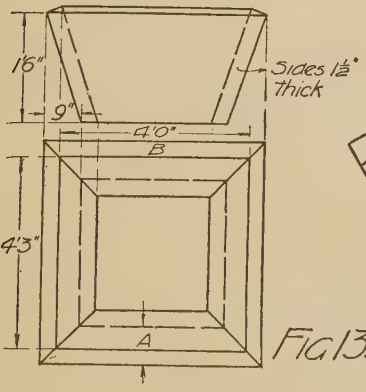
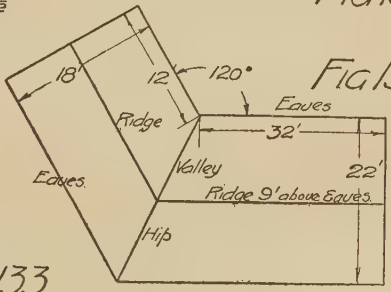


FIG 133

FIG 134



symmetrical. Determine the true shape of the face and edge of any one louvre.

(12) Lay out the inner surfaces of the roof whose plan is given by Fig. 134. Show how the common rafters and purlins fit against the valley, and obtain the bevels for both.

(13) Fig. 135, inverted hopper acting as a ventilator base. It has a uniform slope all round, and is 1 ft. 6 ins. square at the dotted base. Draw its plan if made of $1\frac{1}{4}$-in. stuff, showing the dotted lines,

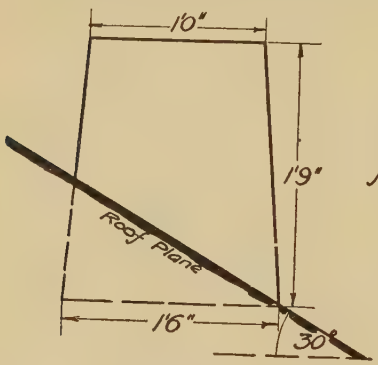


FIG. 135.

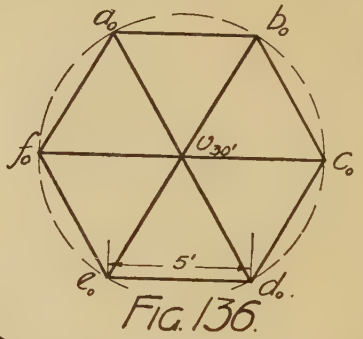


FIG. 136.

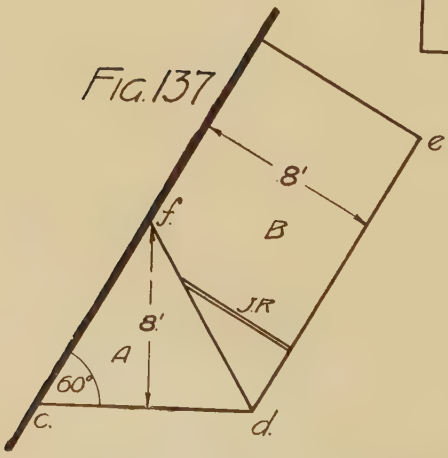


FIG. 137.

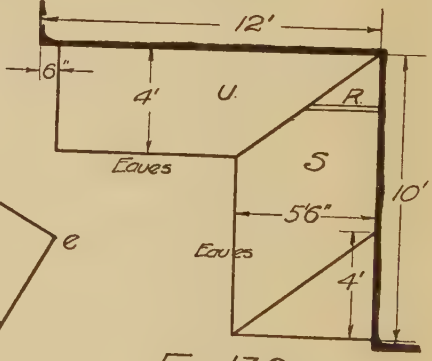


FIG. 138.

indicating thickness and the intersection with the roof. Develop the outer faces in any way you please.

(14) Fig. 136 is the indexed plan of a roof. Its form is determined by the lettering, which indicates the height of any point above the ground

by the number attached to it. Thus f_0 means that F lies on the ground. $V_{30'}$ means that V is 30 ft. above the H.P. Determine—

(*a*) The distance from V to BC, and state which roof member is this length. *Answer*, 30.3 ft., or 30 ft. 3½ ins.

(*b*) The distance from V to C. What use is this?
Answer, 30.4 ft., or 30 ft. 5 ins.

(*c*) The real angle between VC and VB. State its use.
Answer, 9½°.

(15) Fig. 137 is the plan of a building covered with a hipped lean-to roof. The hipped end A and the main roof surface B are inclined planes, whose slope to the horizontal is 30°, eaves CDE being level. Find—
(*a*) True length of rafter DF. *Answer*, 10.3 ft.
(*b*) True shape of surface A.
(*c*) Top surface bevel for jack rafter (J.R.).

(16) The plan of a verandah roof is given in Fig. 138. It rises 3 ft. 6 ins.; the eaves are level. Find the true shape of surface S, the angle between S and U, and mark on your development the angle (or top bevel) for the rafter at R.

Intersection of Roof Surfaces of Constant Slope

In good architectural design the slope chosen as being suitable for the roof of a structure is kept constant for all sections of such roof—or as near constant as possible. The plan of the building "may" cause various heights of the ridges due to differing spans, but this is an architectural advantage. Indeed, in many cases if it so happened that the ridges would be of uniform level the eaves levels would be varied in order to remove the monotonous effect of regularity.

There are many advantages, too, in points of construction when the uniform slope is maintained.

(*a*) Bevels for similar positions are identical.
(*b*) Hips and valleys have the same respective angles, and the special tiles required for these positions would be uniform for each hip and valley.
(*c*) The area of such a roof is very easy to obtain and to check.
(*d*) The pitch of the roof is more easily made to genuinely suit the material of construction.

Now we have seen that whenever the pitches of two intersecting planes are the same, the plan angle is bisected by the hip or valley. Hence, with a uniform pitch this angle is easily determined, but *another* problem arises when the spans differ or the eaves vary in level. We have then a problem in determining the plan outline of ridges, hips, and valleys. For

regular plans this is fairly easy, because we know that "ridges and hips," or "ridges and valleys," must intersect, and the position of the line of intersection is on the bisection of the plan angle between two horizontal lines at the same level in the adjoining roofs. Thus, if two eaves or two ridges meet on a valley or hip they must be on the same level.

Consideration of these points and of previous work should enable you to determine the plans of uniformly sloping roofs if the eaves levels and plan of the building are supplied.

Fig. 139 illustrates such a plan when the eaves are all level and the plan angles between the walls 90°. Think over the

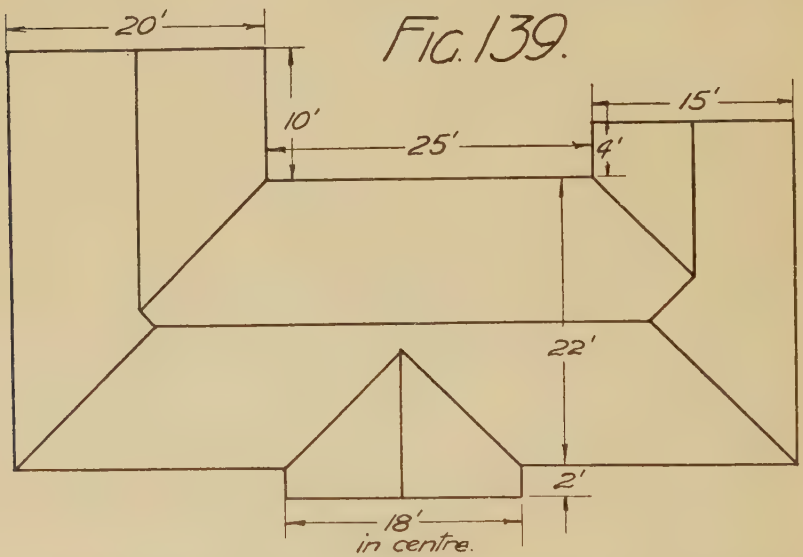


FIG. 139.

above statements, and note that the plans of all hips and valleys are 45° lines. Study the sketches to Figs. 140 and 141, and try to represent Fig. 139 (or parts of it) in a similar way.

EXAMPLES

(1) In Fig. 139 (which you should draw to scale) state which lines represent hips and which lines are valleys. How would you obtain the actual roof area of a covering such as this? Discuss this with your teacher, and refer also to chap. xv. Obtain the area for a constant slope of 45°, given that actual area = plan area $\times \sqrt{2}$.

Answer, 2285 sq. ft.

(2) Figs. 140 to 143 inclusive give incomplete roof plans of houses and buildings all having a constant slope. Sketches of Figs. 140 and 141 are also appended in explanation of the roof intersections. In

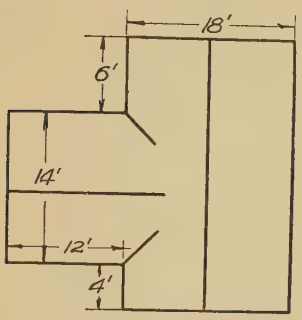


Fig. 140

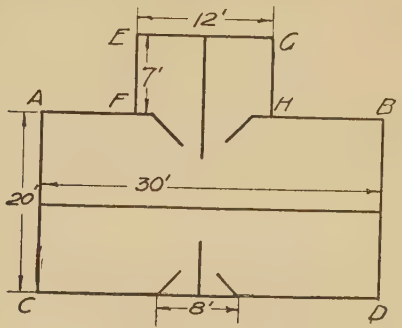
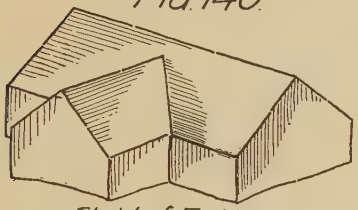
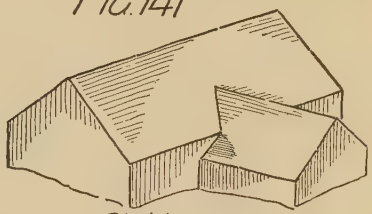


Fig. 141



Sketch of Fig 140.



Sketch of Fig 141.

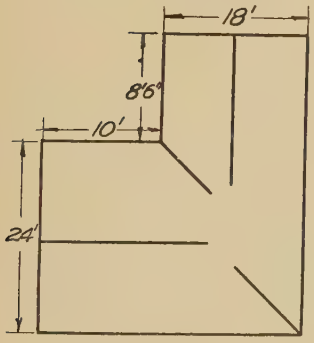


Fig. 142.

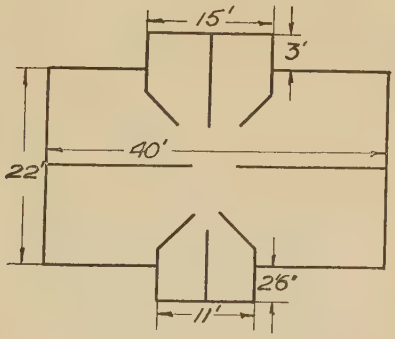


Fig 143.

each case complete the roof plan, altering if necessary any suggested lines, and carrying forward the hips and valleys to their correct intersection at the ridge.

NOTE.—In Fig. 140 the eaves are level. In Fig. 141 the eaves EF and GH are 2 ft. below the eaves at AF and HB. In Fig. 142 the eaves are level. In Fig. 143 the eaves of the smaller cross roof are 1 ft. 6 ins. higher than the "main" eaves.

CHAPTER XI

THE CYLINDER

YOU have already some acquaintance with the cylinder. It is a "solid of revolution" assumed to be described by the rotation of a rectangle about one edge as an axis.

"Any cross section containing the axis" is, therefore, a rectangle double the size of the rotated rectangle.

Every point on the outer edge of the rectangle moves in a circular path at 90° to the axis, and therefore describes a circle whose "radius" = the "breadth" of the rectangle. It thus follows that every section at 90° to the axis is a circle of the dimensions referred to.

The solid generated by the above described rotation is therefore very simple in form, but, if intersected by a plane in "any direction other than those referred to in the above paragraphs," the shape of the section differs from the circle and rectangle, and becomes "elliptical."

It is our business to become thoroughly acquainted with the cylinder and its intersection with other solids, for we meet it on every hand in constructional work, *e.g.*, vertical shafts, as rounded angles, columns, pipes, stair wells, towers, bay windows, etc., and in horizontal positions, as in arches and vaulting. Further, many curved roofs are compounds of cylinders varying in diameter and tangentially connected.

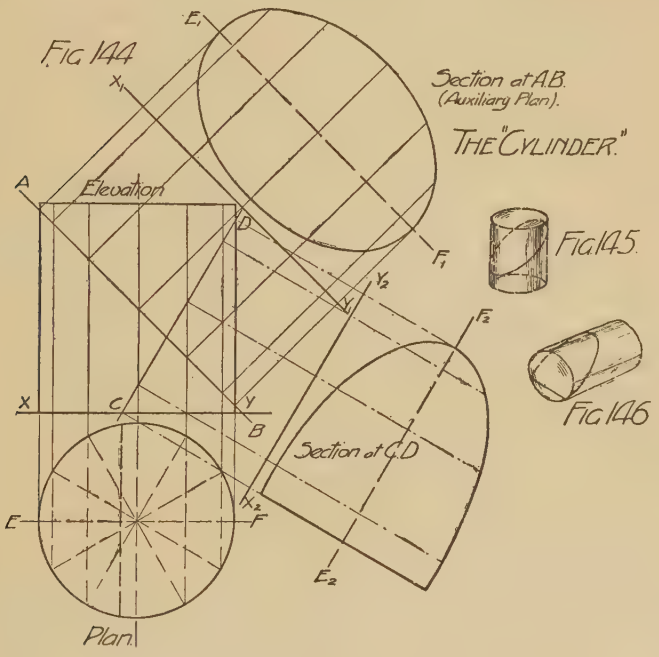
It is well to remember here that we may think of the cylinder as a "solid" mass agreeing with the above description, or as the "surface formed by the boundaries of the rotating rectangle,' or again, as a form intermediate between the two as a thin circular shell, when it is known as a "hollow cylinder."

"Parts" of the cylinder have been considered and projected in vol. i., Figs. 146 and 147, and also at Fig. 153, vol. i.

We shall now consider its "plane intersections." In Fig. 144 we have the projections of a cylinder, stood on its base, and cut by an inclined plane, AB, which passes fully across the solid.

The sketch, Fig. 145, indicates the form of the inclined cut. By virtue of its position, an auxiliary plan will determine the

shape of the section. Because the "plan" of the section is the full circle, we have the two necessary views from which to project the auxiliary plan, viz., the circle in plan, and the edge view of the section, AB. It remains, therefore, to take any convenient points on the plan curve, find their position in the edge view, and measure the position of these points from their plan to XY. Place X_1Y_1 parallel to AB, and clear of the elevation; project the chosen points at right angles, and transfer the measured distances of the plan points beyond X_1Y_1 and complete the



fair curve, which is a true ellipse. The section should be hatched, which is omitted in the figure.

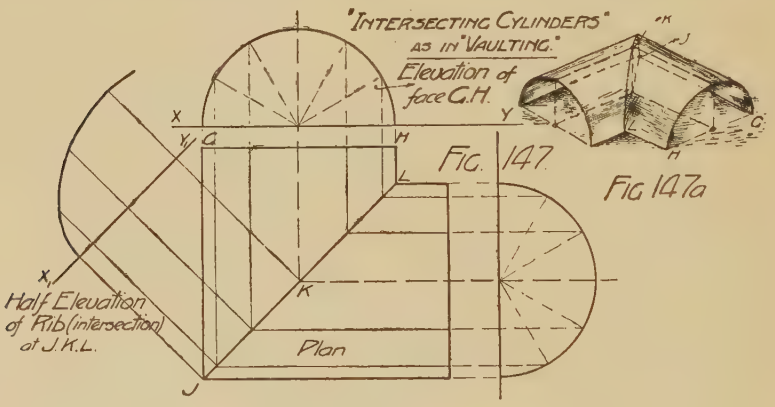
Because of the "symmetry" of this section, a more economical way is to place a line in the auxiliary view equivalent to the centre line EF in plan (parallel to XY). Then, by choosing points in the plan symmetrically disposed about the axis, the measurements of breadth may be economically transferred to each side of the new centre line E_1F_1. (The diagonals dividing the plan circle into equal parts, 12 in number, are obtained either with compasses set to the radius (*see* vol. i.) or with the 30° and 60° set square.)

Your conception of the problem may be assisted by

assuming that the projectors from plan to section line are vertical lines on the cylinder, parallel to the axis, and containing the points of division on the plan. The elevation of the section plane, AB, then indicates the intersection of the plane with these surface lines.

We have said that the section is a *true ellipse, and we must insist that great care should be exercised in drawing the fair curve through the projected and measured points.* Rough, careless work is inadmissible in *any* drawing, and particularly so here. Faults of measurement may be detected and corrected in "lining" the curve.

A second section is illustrated at CD, passing partially across the diameter of the cylinder. Study it with the aid of the sketch in Fig. 146. Then work out the problem fully.



In Fig. 147, a common constructional case is shown, where the "surfaces only" of two semi-cylindrical arches, placed horizontally, intersect each other. Can you see that, on the same principle as two intersecting roofs, when the intersecting surfaces are of the same nature and size, and *not inclined axially* to H.P., their line of intersection is *straight* in plan, as at JL. Examine the freehand sketch of the two cylindrical vaults in Fig. 147*a*, and compare with the plan and two elevations of Fig. 147.

The elevations are alike, and are divided into similar equal parts. If horizontal lines on the cylinder pass through these points along both vaults, notice that they intersect on the diagonal JL. The strong dotted lines meeting on this mitre (or "groin" as it is termed) at K represent the "axes" as well as the highest horizontal lines on the cylindrical surfaces.

It is required for some practical purposes to know the

shape of the "groin" line, and we now see that it is the same as the section made by a vertical plane across the cylinder at JL; hence, a new elevation on a plane parallel to the groin with the heights measured above the XY, and

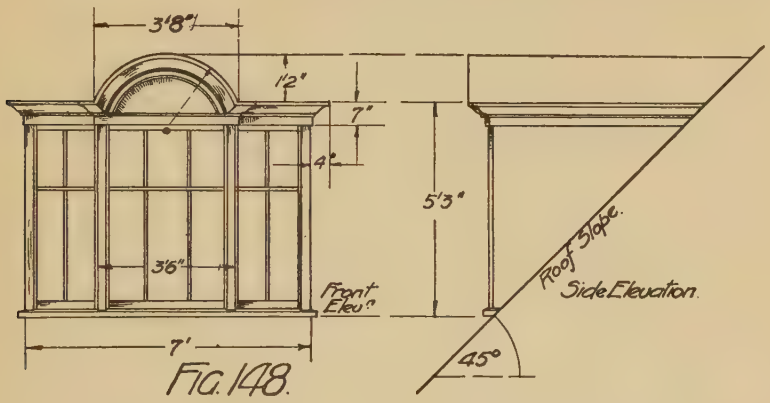


FIG. 148.

transferred to corresponding projectors above X_1Y_1 determines the shape of the groin; again we confirm the fact that an inclined section across a cylinder produces an ellipse.

Complete the elevation of this groin line as an exercise.

EXAMPLES

(1) Taking a cylinder 5 ins. long and 3 ins. diameter, place it in the position of Fig. 144. Assume a section plane at 45° similar to AB in the figure and project the true shape of the section. Now convert this into a "sectional plan" by adding to the view the outline of the cylinder below the section plane.

(2) In your last example assume the upper part of the cylinder (above the section plane) to be removed. Project an auxiliary elevation of the remaining part viewed in a horizontal direction at 60° to the present V.P.

(3) Two arched ceilings (vaults) cross each other at right angles. The ceilings are semi-cylinders 4 ft. diameter with their axes horizontal and at the same level. Determine the outline of the diagonal intersection known as the "groin."

(4) Dormer front with a semi-circular cornice to its roof, Fig. 148. The cornice is fixed on a cylindrical arch rising above the "flat" and continued backwards to intersect the roof as seen in side elevation. Draw the plan from the particulars given in the figure.

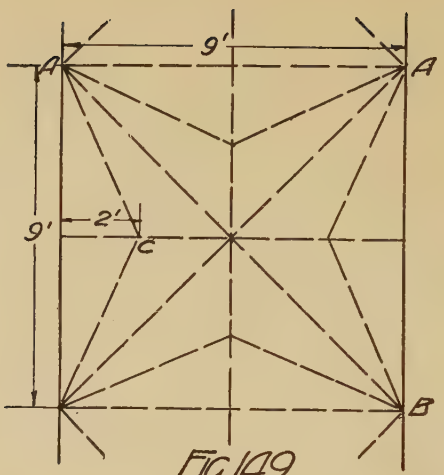


Fig. 149.

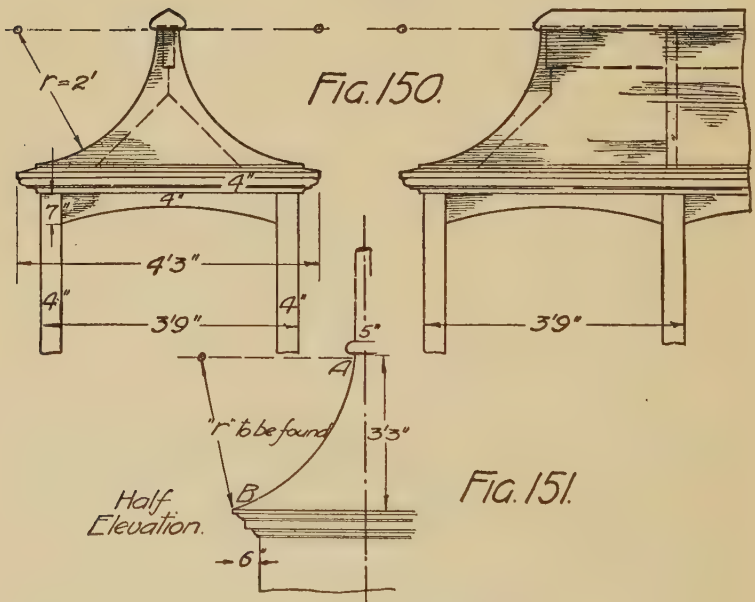
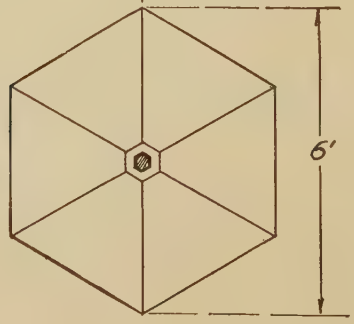


Fig. 150.

Half Elevation.

Fig. 151.



Plan.
Regular Hexagon.

(5) Fig. 149 shows the plan of a cylindrical ceiling constructed in wood and plaster. The section at AA is, of course, a semi-circle. It has been decided to ornament the ceiling with moulded ribs projecting slightly from the "soffit," and the plan of these are given for one "bay." Find the centre line shape of the diagonal curve AB and the short rib AC as contained in vertical planes passing through their plans. Note that the "rib outlines" are the true sections made by vertical planes passing through their plans.

(6) A small porch is covered by a cavetto roof hipped at the front, Fig. 150. The true section of the curve is the same at the front and sides. Given the dotted outline of the ordinary ribs or rafters, determine the true shape of the angle rib, neglecting the thickness of the boarding.

(7) Fig. 151 suggests a hexagonal cavetto roof to a turret. Required, the external outline of an intersection of two roof surfaces from A to B only (exclude the mouldings). What purpose does this outline serve?

(8) Determine the shape of the ribs EG and GF lying on the surface of the cylinder in Fig. 154. Let the diameter of the cylinder be 7 ft., and the inclination of EG to the axis 60°.

The Cylinder—Its Development

We shall now turn our attention to the development of the cylinder and its sectional parts. Its surface, "being produced by the revolution of a straight line, is developable by rolling on a plane." The amount of plane surface traversed, or covered, in one revolution is the developed surface representing the area of the cylinder, and, as you probably know, is a rectangle of dimensions $2\pi r \times l$ (*see* Fig. 152), where r is the radius and l the length of the cylinder. Such a rectangle is "partially" shown in the figure to the right.

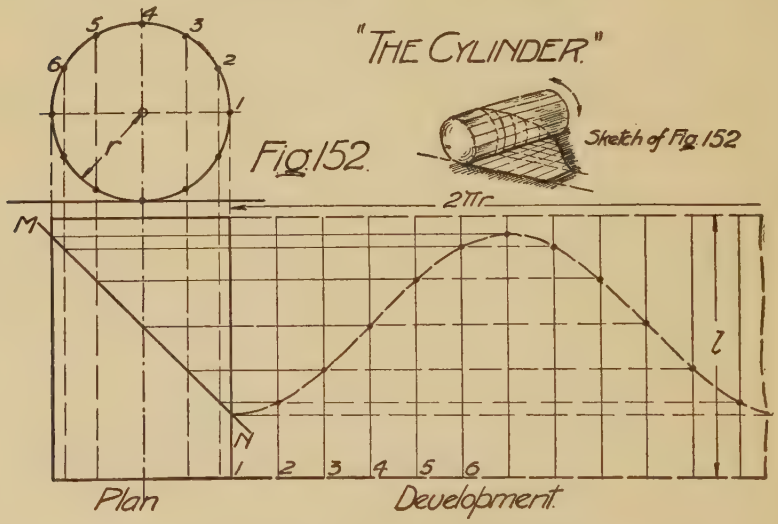
To carry out this development in a drawing we must either calculate the circumference ($2\pi r$) for the "lay out" or take the workshop approximation obtained by dividing the circle into short arcs, such, that when these are assumed to be straight lines instead of curves, the error is very small. This method is clearly indicated in the development shown, the circular divisions 1, 2, 3, etc., being assumed to have the same "arc" and "chordal" length and transferred with the dividers to the development. To obtain greater accuracy the first division of the arc might be subdivided into three or four parts, these transferred to the straight line, and the "sum" of these smaller divisions repeated until the circumference is fully developed.

Now let the cylinder be cut by a vertical plane MN. For

many constructional problems we need to know the developed surface of the cylinder with the outline of the intersection made by this plane. When a cylinder is rolled, every point on its surface moves with the cylinder in a plane at right angles to its axis; hence we can trace the paths of any selected points when unrolling.

From the plan and elevation we know the position of certain points on the section plane MN, determined by this line cutting horizontal lines on the plan of the cylinder, whose end views are the points 1, 2, 3, etc., in elevation.

On the full development, therefore, mark the positions of these horizontal lines across the space, parallel to the axis.



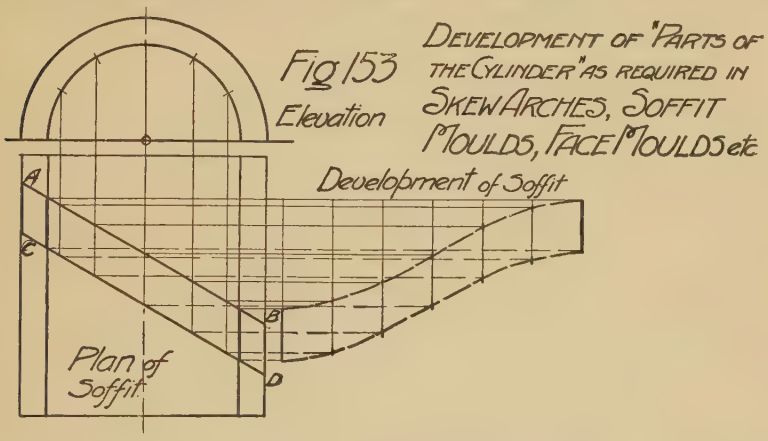
Assume that the cylinder rolls, as shown in the sketch above the figure, leaving the impress of the cut on the plane.

You can obtain several special points of contact along the section line, by projection, noting the sequence of the numbers, and mentally seeing each point rotate to its position. These points are clearly marked, and the fair curve through them is the outline required.

The practical application of this principle in architectural construction is very wide, and we can only find space to take a few typical examples to show the value of it.

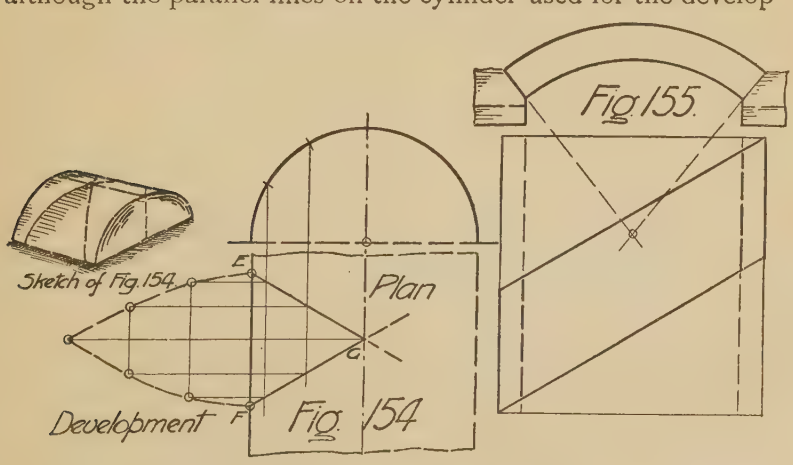
In Fig. 153 is the plan (projected up) and elevation of a semi-circular arch. Suppose we did not desire to use the whole of this rectangular arch, but only a portion of it to span from side to side on the slope, as shown by the thick lines.

When you reach more advanced work you will find that we need to know the shape of the "soffit" enclosed between the vertical planes AB and CD, in order to place the joints and determine the sizes of the stones. Consider the inside of the



hollow cylinder in elevation, and make the stretch-out as you did in Fig. 152. You obtain "two" curves instead of one, and of "half" the circumference.

Notice that these "curves of development" are *not parallel,* although the parallel lines on the cylinder used for the develop-



ment are of equal length between the vertical planes AB and CD. When cut out and bent over the arch, the surface between the curves *appears* parallel.

Another application is shown in Fig. 154. We often form

roofs of "parts" of cylinders intersecting each other, so that one section of the "whole" roof might be a part of a cylinder such as the triangular outline in the plan, EFG. Required, the shape of the surface enclosed. If you have any difficulty in deciding its form mentally, study the sketch of Fig. 154; the curved "wedge outline" is the part to be developed. In the figure it is seen to consist of two parts of curves similar

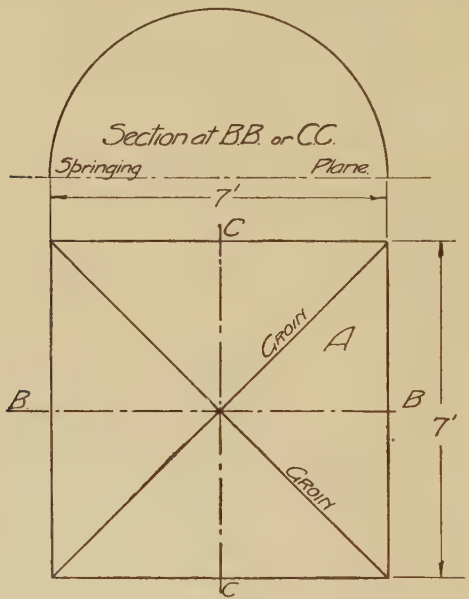


Fig. 154.a.

to those of Figs. 152 and 153, and obtained by the same process of unrolling the curved surface.

Refer now to Fig. 156 which illustrates another application of the unrolling of a cylinder with outlines of intersections made by other cylinders. It represents, in line diagram, a vertical cylindrical wall with a bull's-eye window in it and a semi-circular arch. *These figures are "circular" and "semi-circular" in elevation.* Find their true shape on the developed surface.

Applying the previous principles choose convenient points on the elevation curves (equidistant ones are best). Find the plans of these and stretch out the circumference of the plan with the points 1, 2, 3, etc., upon it. Place these points by stepping off short lengths with your dividers, marking specially 1, 2, etc. The longer lengths 3-4, 4-5, might be taken at two steps. In elevation the points appear to move horizontally

(at 90° to the axis), and pairs of projectors determine their developed positions; fair curves complete the figures.

Observe that these true shapes are not ellipses; they are not formed by plane intersections of the cylinder.

The above process is identical in principle with the development of "face moulds" in arches of double curvature.

In Fig. 157 and its accompanying sketch we have an illustration of the cylinder in its compound form, applied to an "ogee" roof. Each "side" of the roof is composed of two circular arcs, and the method of finding these is demonstrated on the right. Each of these arcs is the end view of a portion of a cylindrical surface, hence the surface may be unrolled on the same principle as before.

Notice further, that the plan of the roof is a square and the surfaces are all of the same curvature, producing intersections at the diagonals of the square. The process should be fairly obvious, but is briefly summarised thus: Divide the elevation curve into suitable parts. These need not be equal, but are advisedly so between the ends of the different arcs. Transfer in proper order to the centre line of the development which is projected from the centre of the plan at 90° to the eaves. Find the plans of the horizontal lines whose end views are the points of division on the elevation curve; these terminate on the angle ribs in plan and are projected horizontally to meet their respective sections of the laid out strips or bands of roof surface. The fair curves through the projected points complete the development.

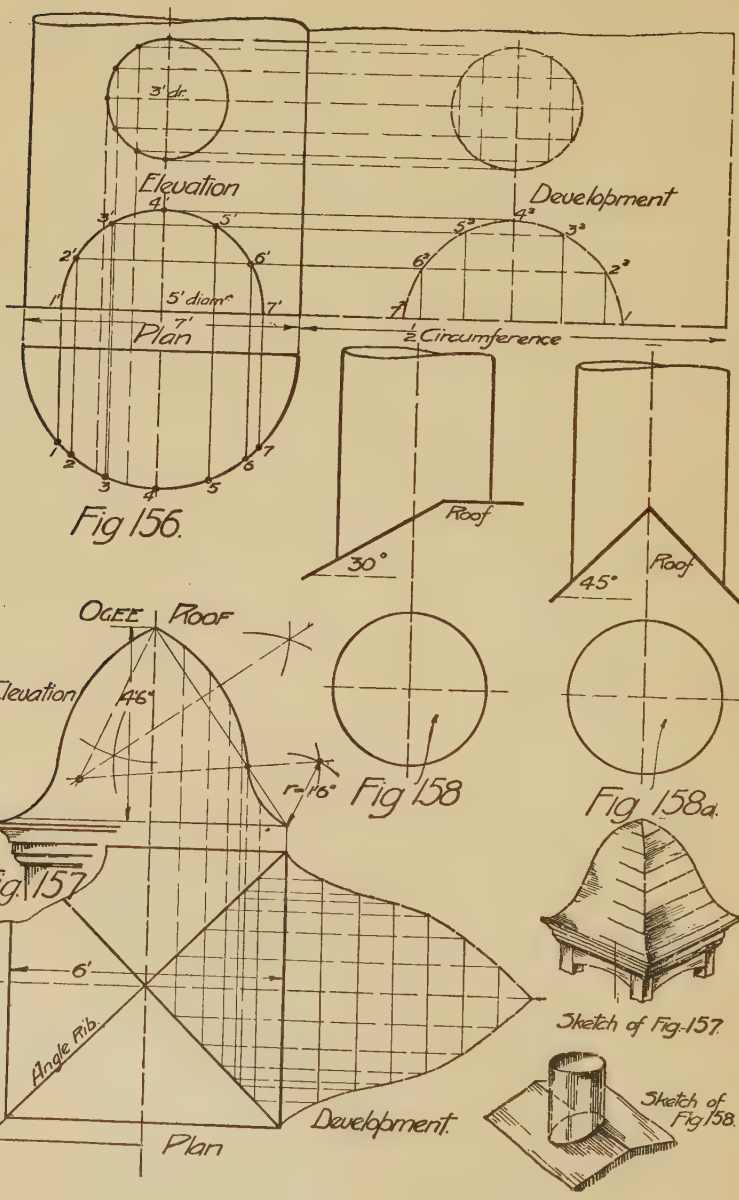
EXAMPLES ON THE DEVELOPMENT OF THE CYLINDER

(1) Set out the conditions of Fig. 152, the cylinder to be 5 ins. long and $3\frac{1}{2}$ ins. diameter. Let the plan MN pass through a point $\frac{3}{4}$ in. from the right-hand angle of the plan, and passing to the left downwards at an inclination of 45°. Develop the segment of the cylinder intercepted to the left of the section plane.

(2) Let Fig. 153 represent an arch 10 ft. span and 12 ins. deep on the face. Let a skew arch in the plan be similar to ABCD where the slope is 60° to the abutment, and AC = 5 ft. Obtain—

(*a*) The shape of the intercepted soffit;
(*b*) The shape of the intercepted extrados.

(3) Plan and cross section of a square-domed ceiling, composed of parts of cylinders, Fig. 154*a*. These intersect along the diagonals in plan. Develop the surface of the "gore" marked A lying between the two mitre lines or "groins."


3' dr.
Elevation
Development
5' diam^r
Plan
½ Circumference
Fig 156.
Roof
30°
45°
Roof
Fig 158
Fig 158a.
Ogee Roof
Elevation
45°
r=16"
Fig 157
6'
Angle Rib
Plan
Development
Sketch of Fig. 157
Sketch of Fig 158.

(4) Fig. 155 shows a small skew segmental arch in plan and elevation. Its span is 6 ft., angle of inclination to the abutment 60°, thickness of arch 10 ins., and radius of curvature 4 ft. 6 ins. Obtain the shape of the developed soffit between the full lines which are 3 ft. apart measured parallel to the supports.

(5) Work out the figures of diagram 156 completely to the dimensions given. Can you suggest a method for curtailing the space taken up by this solution?

(6) Refer to Fig. 170. Design an elevation similar to this figure. Suppose it to stand over a square plan. Develop the roof surface and find the shape of an angle rib.

(7) Fig. 158 illustrates a shaft of 3 ft. diameter passing through a roof at the junction of the slope and a flat. It is to be made in sheet metal to fit the surface. If the flat enters 9 ins. horizontally into the cylinder, determine the shape of the cylindrical surface at the base. *See* also sketch to Fig. 158.

(8) A similar shaft, 2 ft. diameter, intersects a V roof at the ridge, as per Fig. 158*a*. Develop the part of the shaft above the roof planes for a height of 1 ft. 6 ins. above the ridge.

(9) Refer to Fig. 152 and to the dimensions of problem 1. Find the area of the intercepted surface of the cylinder to the left of the section plane. Observe that a consideration of your book development at Fig. 152 will easily prove the following to be true : —

$$\text{Area of oblique cylinder} = \text{circumference} \times \frac{\text{longest length} + \text{shortest length}}{2}.$$

Answer, 27½ sq. ins.

(10) Obtain the area of the soffit of the skew strip of arch in problem 2. Area = length parallel to axis × ½ circumference. Satisfy yourself of the truth of this from the development of Fig. 153.

Answer, 78¾ or 78.55 sq. ft. (approx.)

CHAPTER XII

THE CONE—ITS SECTIONS AND DEVELOPMENT

THE cone—or some part of it—is embodied in many examples of building construction. It may be found in roof turrets, splayed window linings to circular openings, framed canted seat-backs, and joints in masonry, domes, and niches.

We are specially concerned with its "form," curves of plane intersection which may be projected from it, and the development of its whole surface or such parts as it may be desirable to employ for our purpose.

Previous study has probably made you acquainted with its form, and the mode of its generation as a geometrical solid.

In its simplest form you will recall that its shape is determined by the rotation of a right-angled triangle about its perpendicular, the hypotenuse generating the inclined surface known as a "conical surface."

Think of the generating triangle as standing in a vertical plane; let it produce a right cone with its axis vertical (Fig. 159). Observe—

(*a*) From the process of generation the horizontal section anywhere (at 90° to the axis) is a circle whose diameter is determinable from the elevation.

(*b*) A vertical section through the vertex, containing the axis, will be a triangle of twice the area of the generating triangle.

(*c*) A straight line may only lie on the conical surface completely, when it occupies one of the positions through which the hypotenuse of the generating triangle passed, during rotation.

Obtain a right cone, examine it again and satisfy yourself of the correctness of the above statements. Then proceed to prove further that any section of a cone, other than those mentioned at (*a*) and (*b*) above, cannot be either a triangle or a circle. (A "sectional" cone is required here.)

Any plane section of the cone which cuts two generators

on opposite sides of the solid produces an "ellipse," when the section is inclined to the axis. We shall study such a section here in detail.

Let us show that if any such cone, as, for instance, a roof, be projected above a plane sloping roof and intersected thereby, the outline of the junction will be an ellipse, and the form of the curb at the junction may be determined by direct projection.

Fig. 159 is the plan and elevation of a cone intersected by an inclined plane MNO, at 90° to V.P. (Observe that the

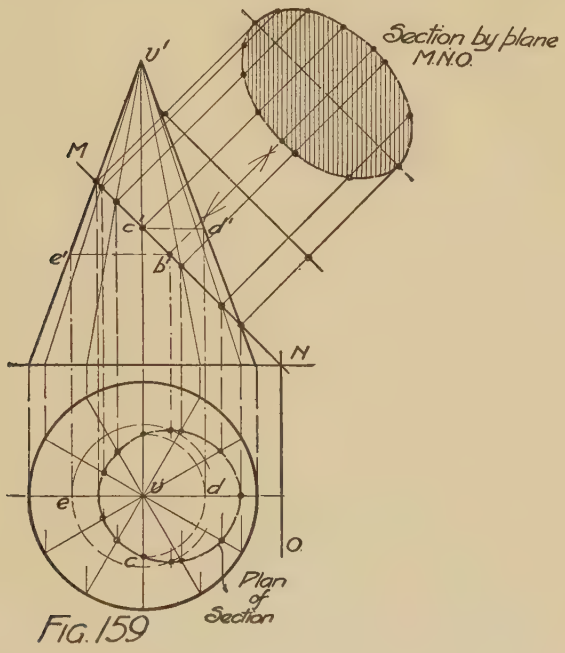


FIG. 159

section does not pass through the base of the cone.) The true shape of this section is required necessitating an auxiliary plan.

Recall the first necessity when an auxiliary view is to be projected, viz. a plan and elevation of the outline to be solved. In this case we possess "one view," the line MN which, in elevation, is the "edge view" of the sectional surface. We must therefore obtain its "plan."

Draw the plans of several equidistant generators (positions of the hypotenuse of the triangle which generated the cone). These, twelve in number, radiate from the plan of the vertex, and are obtained with the 30° and 60° set square or compasses as in previous problems.

Project the elevations of these lines by finding their positions on the H.P. and joining these to the elevation of the vertex. Notice that, except for the outside positions of the generator in elevation, each inclined line represents "two" elevations. Drop projectors from M and N to the plans of these lines, and also from the points of intersection of MN with the remaining lines to their respective plans. Follow these carefully on the drawing.

You will meet a difficulty with the middle line opposite the axis, because the plan and elevation are both vertical.

Now remember that the elevation of the cone is the same from whatever position you view it, and if a horizontal section be taken across the cone through the point c^1 in elevation, it would cut in a circle whose diameter is the distance between C and the opposite point on the back of the cone.

Therefore project horizontally from c^1 (*see* dotted line to the right, c^1d^1), drop d^1 to the plan at d, and transfer the length vd, to the vertical centre line on each side of v. This determines the two points on the plan curve which could not be directly projected.

The plan outline may now be drawn through the points already obtained, or the points may be used to project and measure from, *without drawing the fair curve*, for it is of no real use.

Before doing this, however, we prefer to find the position of the points in plan at which the curve reaches its greatest width at 90° to XY. Bisect MN at b^1; b^1 is the elevation of the "minor axis" lying behind the point on a horizontal line through b^1. Project horizontally from b^1 to e^1, and from e^1 to e in plan. Describe a circle through e, with v as centre, and project b^1 down to it, intersecting at two points. These are the ends of the minor axis in plan, and the curved figure (an ellipse) attains its greatest width here.

For the true shape draw an auxiliary plan of the section projecting from MN at 90°. Choose any XY as a base line clear of the elevation, and transfer distances in front of XY outwards from the "new" XY, or, as an alternative, take a base line such as the dotted centre line of the section, and measure the symmetrical distances on each side of it from the similar points on each side of the corresponding centre line in plan. MOST ACCURATE DRAUGHTSMANSHIP IS NECESSARY FOR A REASONABLE RESULT.

It is now necessary for you to give second thoughts to the same problem, because there is another method in common use for arriving at the results, viz., by determining the plan outline from the property that "every horizontal section is a circle."

Take a series of horizontal sections "lying within the elevation of the cutting plane," and construct the intersections of the plans of these horizontal circles with the plane MNO; thus, take "any" horizontal section, project its diameter to the plan, and draw the circle. Note the two plan points on the circle immediately below the elevation at the "cut" of MN and the chosen section. Repeat this for several sections, and complete the problem as an exercise. (No illustration.)

Development of the Cone

Our next concern is the method of developing the cone. Choose the same right cone as in Fig. 159. If whole, we can see at once what form it takes by rolling it on a horizontal plane. (You should have read the references to this at p. 78, Fig. 117, and have seen that its development is analogous to that of the pyramid with an infinite number of sides.) Should you have omitted this introduction, return to it, and read carefully.

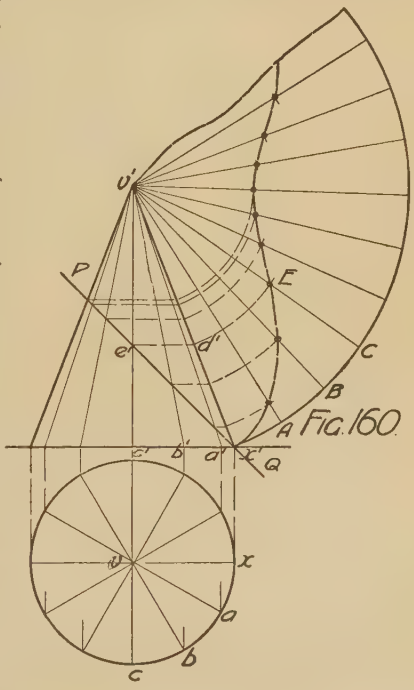


Fig. 160

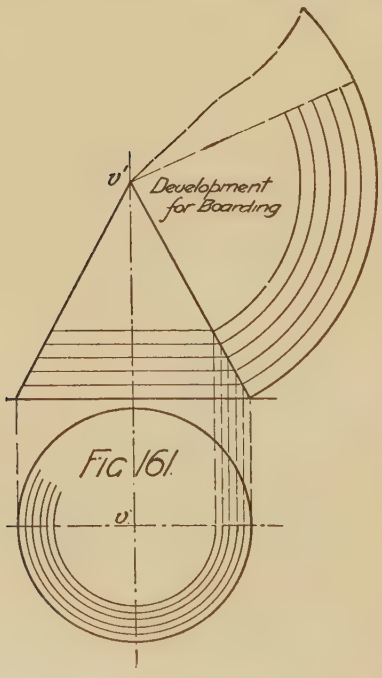
Now observe that part of a similar development is reproduced at Fig. 160. We desire to show what part of the surface area of the cone would be exposed above the roof if severed by an inclined roof plane. Let PQ be the edge of the roof plane. Draw plan and elevation of the generators as before, then note that if the generators be shown on the development at the correct distances apart so that *ab* in the plan curve equals AB in the development, we can determine the shape of the upper portion of the cone by solving the position of the points of intersection on the corresponding generators in the development.

The problem is, then, to find the true length of each intercepted generator from the vertex.

All generators being the same length, and their real length

seen at the outside lines in elevation, project each in the section plane PQ horizontally to the outside line, as from e^1 to d^1. Transfer d^1v^1 to E with the compasses. E is the developed point. Perform the same operation for each point and join the developed points by a fair curve. A little over half this development is shown to exhibit properly the form of the curve.

Now observe the practical value of these and similar developments. Should we require to cover the surface with boarding by cutting the boards in the form of the tapered strips into which our development is divided, these, cut to the correct bevel on the edge, may be forced round the cone and nailed down so as to correctly and perfectly cover it, or any exposed part we may have developed.



Any strip may be subdivided or jointed in its length to suit the framing on which it is laid if the sum of the sections makes up the complete development of the surface to be covered. Bear in mind, also, that your development must be that of the final "surface covering," not that of the framing on which it is to be laid.

Lead or copper covering is dependent on the same surface development, and if one part is to overlap another it is allowed for in "cutting."

Other methods of subdividing the developed area into strips are possible, and one good practical method must be given here. It is suitable for covering framing, the "main" of which lies in the direction of the generators of the cone.

This "method" consists of dividing the area into parallel "zones" or rings, so that the development becomes a series of parallel circles. Fig. 161 shows the method. Horizontal rings of division are shown clearly in plan and elevation; then, because the base ring of the "development" has its curvature known, and all other rings must run parallel to it,

we can proceed as follows: Choose the "width" of board, set it up on the elevation along the extreme generator, develop the whole (or half) of the cone, and describe the parallel curves on the development by projection from the divided generator.

Observe that in practice neither the plan nor elevation of the joints in the boarding are of any use. They are required here to make clear the process and the position they are to occupy in the boarded roof.

It is now easy to understand that if any cone or "part" of a cone requires developing the foregoing principles must be applied. The student's difficulty will most probably be, to recognise the "part" and its relation to the whole cone in a practical case.

One of the best illustrations occurs in the "head lining" to a circular-topped window opening with splayed linings

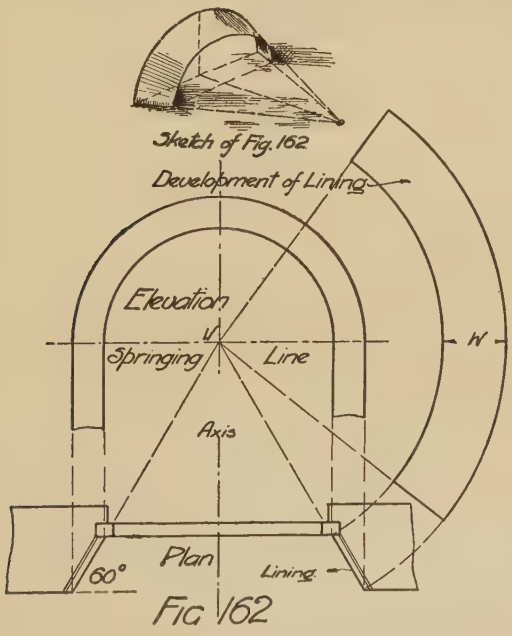


FIG 162

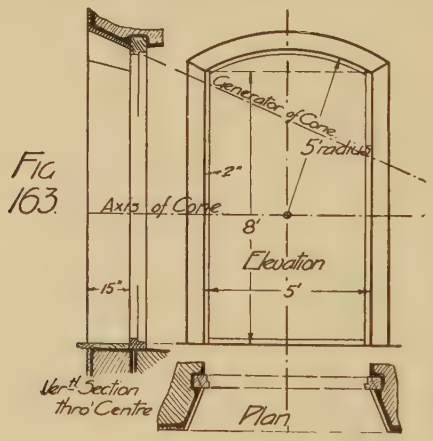
between window frame and plaster. Fig. 162 illustrates the conditions. Its plan and elevation shows that the "splay" is at a constant angle, because carried uniformly round the head. Observe also "the face edge of the lining and the back edge against the window frame are *both* circles." Is not this a first condition of the cone? Study it and make sure. It is, however, a horizontal cone, not a vertical one, and only "part"

of half the cone exists. This part is a strip exactly corresponding to one of the "boards" of Fig. 161. Study the sketch of Fig. 162 and compare.

To obtain the plan of the cone showing the extreme generators, take the plan of the splayed jambs at the springing and produce to meet on the centre line at V, which is the vertex of the horizontal cone. (In this illustration it is a mere coincidence that V lies on the springing line in elevation.)

Now develop "that strip of the cone which is the breadth of the lining W." The distance round the developed lining at the outer edge is the circumference of the elevation at the *same* edge. To make this quite clear you should mentally compare Fig. 162 with its sketch. The lining in the latter is shown in skeleton form, representing the inner face of the lining.

Now pass on to Figs. 163 and 166. These are two windows with segmental tops, but the former is finished with a "splayed



lining of the same pitch all round," while the latter has splayed sides and a "segmental head lining *square* to the face of the frame." The head lining of Fig. 163 is, therefore, part of a cone, while the head of Fig. 166 is part of a cylinder. The methods of solving both problems will be explained here in order to avoid any confusion between the two forms.

Fig. 163*a* illustrates the application of principle for Fig. 163 in single lines. A similar head lining is drawn, but with more curvature. Being *part* of a cone, our first operation is to *complete* it. In elevation, continue the curves of the lining down to the level of the axis, and the splay of the lining in section down to the same level. The section then shows the

length of the generator of the whole cone, and the elevation curves show the lengths to be measured round the "developed curves" which we shall obtain from the section. Draw these latter at ABC, DEF; take the line VB anywhere sufficiently distant from the section, and use this as a centre line from which to measure the circumference of the lining "edges."

Divide the "elevations" of these curves into convenient parts as shown, and set them off along the "developed curves" from B and E respectively to AC and DF. Join AF and CD; then the figure ACDF is the developed lining. Observe that the "ends" of the curved strip of lining "are not radial to

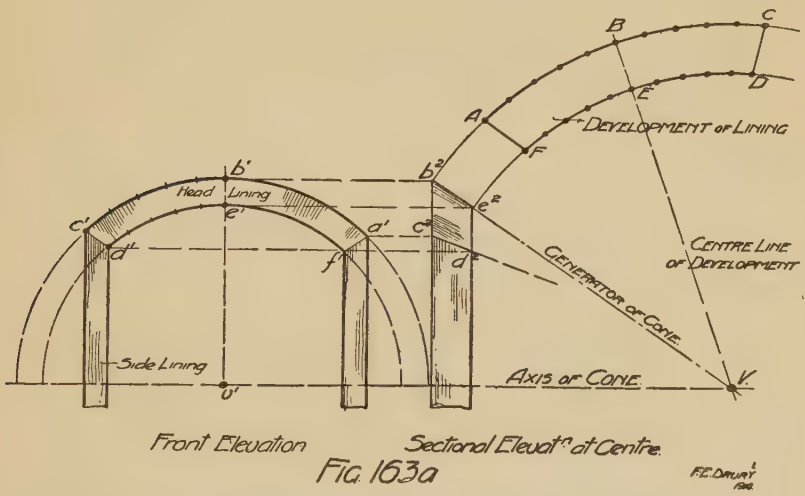


Fig. 163a

the centre V," because of the intersection of top and side linings not lying on a generator through v^1 in elevation. This is our reason for *measuring the curves from the centre.*

Fig. 166 needs the head lining of the proper curved length in elevation merely bending and fixing in position to the required radius, but the side linings here need special treatment. Because *they* are splayed and the *head* is cylindrical, the outer edge of the lining is shorter than the inner one, causing the end of the side lining to be curved and not truly seen in elevation. It is, therefore, necessary to develop the side lining.

Assume one or two intermediate vertical lines to lie on the face of this lining between the outside edges and at uniform distances apart. The end view of these are "points" in the plan lying on the "face" of the lining. Divide the plan and draw the elevations of the chosen lines, then revolve the plane of the lining into a suitable position for view, viz., parallel

to the wall. The "widths" of the several strips of lining are now truly represented, and if the length (height) of each line be maintained from the level of the window board, a fair curve through the "head points" will determine the required shape of the side lining to fit against the cylindrical top lining. Study the right-hand part of Fig. 166 where the thicker lines show the lining and head curve needed in the solution. We shall now embody these principles in the following examples.

EXAMPLES ON SECTIONS AND DEVELOPMENT OF THE CONE

(1) Fig. 164 shows the edge view of a roof plane pitched at 38°, with a conical roof rising through and above it. Let the base of the cone be 6 ft. diameter, and its height 10 ft., with the edge of base 12 ins. from the eaves of the roof. Find the true shape of the section made by the roof.

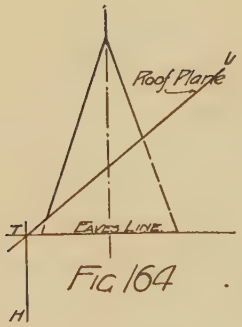


Fig. 164

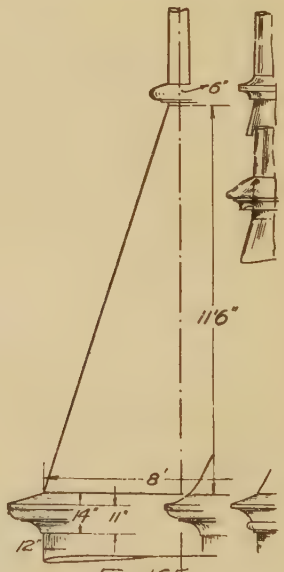


Fig. 165

(2) Develop the whole surface of the conical roof of the last example (Fig. 164), and show by a strong dotted line the outline of the part cut off by the roof slope. Shade the part which stands above the roof surface.

(3) Fig. 165 is the elevation of a conical turret roof, with an 8-in. diameter flagstaff rising from the top. The roof surface is, therefore, a cone frustum. Develop the surface of the frustum, and calculate its area in square feet. *See* note at head of p. 116. (For calculation *see* below.) *Answer*, 164.3 sq. ft.

NOTE.—In your calculations you should have learnt that the slant area of a cone is the $\frac{\text{"slant height"} \times \text{"circumference at base"}}{2}$. You should be able to verify this from Fig. 117.

Let the slant height be s, the radius at base r, the circumference at base $2\pi r$, then area $= \frac{\frac{1}{2}\pi r s}{\frac{1}{2}}$. Also s, being the hypotenuse of a right-angled triangle of base r and height $h = \sqrt{r^2+h^2}$, therefore we may write the area (A_c) thus :—

$$A = \pi r s = \pi r \sqrt{r^2 + h^2})$$

The area of the "frustum" of a cone (A_f) may be shown to be—

$$\frac{\text{"slant height"} \times [\text{base circumference} + \text{top circumference}]}{2}.$$

Let top circumference be $2\pi r$, and base circumference be $2\pi R$, and slant height s. Then—

$$A_f = \frac{s \times (2\pi r + 2\pi R)}{2}$$
$$= \frac{2\pi s\,(r+R)}{2}$$
$$= \pi s\,(r+R).$$

This also should be clear from the frustum development of Fig. 117, on the left.

(4) A right conical roof to a turret 4 ft. diameter has a rise of 4 ft. 6 ins. Calculate the nett area of lead required to cover it. What would be its weight at 6 lbs. per ft. sup., with an allowance of 8 per cent. for laps and waste? *Answer*, 30.95 sq. ft.; 1 cwt. 3 qrs. $4\frac{1}{2}$ lbs.

(5) Find the area of copper tiling in a turret roof of conical outline. The base diameter is 12 ft., and the vertical height 14 ft. A finial, 10 ins. diameter, springs from the cone at the top, reducing it to a frustum.
Answer, $A_f = 303.8$ sq. ft.

(6) The lining to the head of a semi-circular topped window is 4 ft. 4 ins. diameter at the plaster line, covers 11 ins. of jamb between face of plaster and frame, and splays 3 ins. in this width. Determine the true shape of the lining.

(7) Construct the true shape of the head window lining of the example in Fig. 163, which you should draw to $\frac{1}{2}$-in. scale or larger.

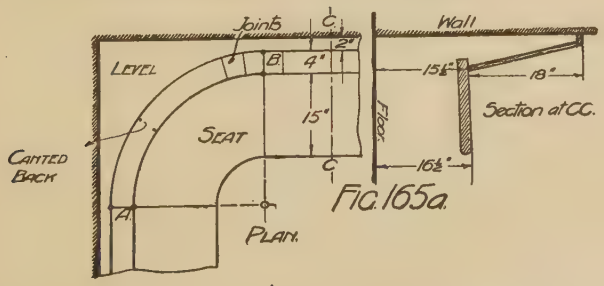
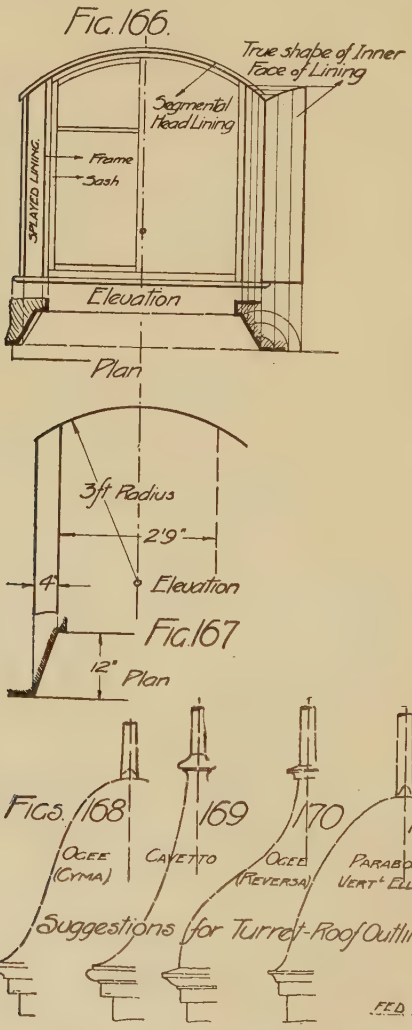


Fig. 165a.

(8) Fig. 165*a* is the plan of an angle in the fixed seating to a schoolroom. The back slopes 3 ins. in a height of 18 ins. Find the shape of the developed back between A and B, and mark a board suitable for repeating round the angle so as to cover it neatly.

(9) Obtain the true shape of the splayed lining for the side of the window illustrated in Fig. 167. The head lining is segmental, and at right angles to the frame.

NOTE.—In connection with Fig. 165 are illustrations of mouldings suitable for certain positions in the structure, *e.g.*, at the "eaves" and "finial junction." You are not to draw all these, but select the type you prefer while noting the features of the rest.



In Figs. 168 to 171 inclusive are suggestions for some forms of roof employed in covering ventilating turrets and similar ornamental features. As an exercise, any one of these might be selected to cover a square, hexagonal, or octagonal plan, and the roof surfaces and angle ribs developed. Refer to Fig. 157 and the description therewith for the explanation of principles.

The Parabola

Before leaving the consideration of the cone we must note the particular plane section which is known as the "parabola." It is produced by a section plane severing the cone parallel to a generator of the solid, as illustrated in Fig. 172. Two such sections are constructed, produced respectively by planes MN and RS. Notice what a difference occurs in the form of the curve due to the different position of the section planes, yet the resulting curve may be proved to possess similar properties and conform to a similar mathematical law.

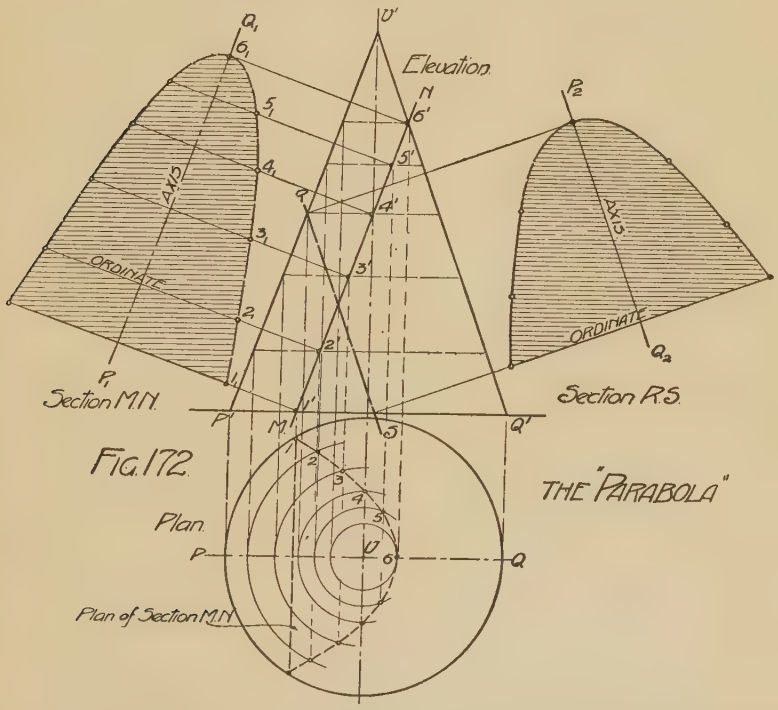


Fig. 172.

Our interest in this curve arises (*a*) from the possible intersection of two constructive solids, one being conical and the other having a plane surface, and (*b*) the fact that many curves representing graphically the variation of one value with the change of another are parabolas.

We shall first obtain the curve by construction as a definite section of a chosen cone, and later, by applying certain constructions based on mathematical properties of the curve.

In obtaining the section of a cone parallel to the generator

(or in any other position which does not produce an ellipse), it is better to employ the property of constant form of horizontal section, viz., circular.

This is applied to the left of the cone in Fig. 172 for the section at MN. A plan of the section has first been constructed by taking a series of horizontal section planes between M and N, finding the diameter of each circle made by these planes, and drawing a part of its plan. Now determine where each point, as 1^1, 2^1, etc., on the elevation, lies on the corresponding circle in plan. A fair curve through the points with a straight line across the base completes the plan of the section, and a new plan of this symmetrical figure projected from the edge view in elevation and measured from PQ on each side gives the true shape of the section. P_1Q_1 is called the "axis" of the symmetrical curve, and any line across the area at 90° to the axis and bounded by the arc is called an ordinate. A similar section has been determined at RS, but the working is entirely omitted. You will do this as an exercise.

The mathematical properties of the parabola are only of interest to us so far as we can intelligently use them. We may need, for instance, to draw a curve of some specified width at the base and of some definite rise, the curve following a parabolic outline. If this were to be constructed in wood or stone it would require joints making across it normal to the curve at some chosen points. We also need the curve to represent variation of bending moment in a beam carrying a uniform load. Hence our uses will at least be two in number, viz.:—

(*a*) To draw the curve to given dimensions.
(*b*) To construct a normal at any point on the arc.

You will see later that the "normal" always involves a "tangent."

Figs. 173 and 174 show two methods of determining the curve to given dimensions.

Method 1.—(Fig. 173, the "radial" method). Let NO be the base ordinate, and 4 V the rise of a parabolic arc, the vertex to be at V. Construct a rectangle $M4_0ON$, divide the base into any number of equal parts, say 8, and the rise into half that number. Refer to the right half of the figure; these points of division are marked 0, 1, 2, etc., and $0, 1_0, 2_0$, etc. Draw a series of lines parallel to the axis V4 through the horizontal divisions, and a series of radials through V from $3_0, 2_0, 1_0$. The intersections of 1.1_0, 2.2_0, etc., give points in the

curve, and o, V are also points in the arc. Repeat for the other side and draw the fair curve.

Method 2. — (Tangential method, Fig 174). This consists of the construction of a series of tangents to which the fair curve may be drawn.

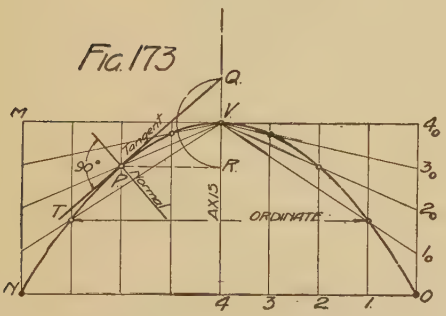


Fig. 173

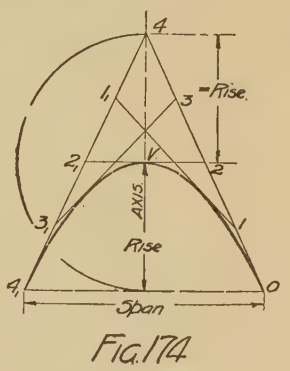


Fig. 174

The span and rise are set out as before, and the latter repeated on the opposite side of V at V4. Join 4.4_1 and 4.0 forming an isosceles triangle on the span and of twice the rise. Divide 4.4_1 and 4.0 into any number of equal parts. Number them in opposite directions as per Fig. 174. Then 4.4_1, 3.3_1, 2.2_1, 1.1_1, 4.0 are all tangents to the curve, and a fair curve touching these lines centrally at each segment on the "interior" is the parabola required.

This is an excellent method and requires "few" points; in fact, it is always unwise to select too many points for a freehand curve after you have become acquainted with its form and characteristics.

To draw a tangent at any point on a parabola, such as point P, Fig. 173, drop a perpendicular from P to the axis at R. Transfer VR to VQ. Join QP, which is a tangent at P. The normal at a point on a curve is always at right angles to a tangent at the point, hence our normal is determined.

This construction is based on the property of the parabola (which is easily verified) that "every tangent to a parabola intersects the axis produced, at a point the same distance from V as the distance of the point of contact from a tangent at the vertex.' The statement is cumbersome, but by reference to the figure you will see it defines that VQ = VR for the point of contact P.

It may be proved that we have used this property for

determining the tangents in Fig. 174. A smart student will probably see the application.

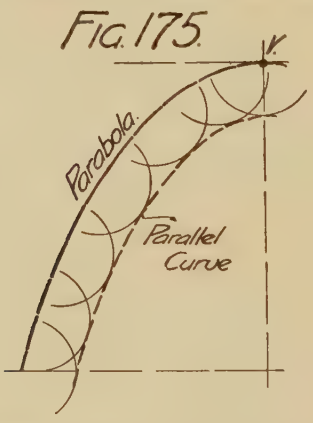
There is another very important "sectional curve of the cone" formed by a plane severing the cone parallel to its axis and known as the "hyperbola." Important as this curve is in some branches of industry its occurrence in building is so infrequent that we cannot find space to include it here. Being constructed on the same basis as a parabola, the student should be able to project it from the cone if desired.

EXAMPLES ON THE PARABOLA

(1) Take Fig. 172 and construct the parabola of section due to the cutting plane RS, or "any plane parallel to v^1q^1." (*See* Example 2 before commencing.)

(2) Prove that the construction of Fig. 173 agrees with that of your projected section by applying the "radial" method there shown to your projected drawing. Work to a large scale for this reason.

(3) The intersection of an inclined plane with a cone resulted in a parabola 4 ft. across the base ordinate, and 3 ft. high. Set out the curve by the method of ordinates and radials, Fig. 173. Draw a curve within



this and 4 ins. from it. These parallel curves represent a mould for cutting a timber curb, on which rafters will fit. A joint is required 1 ft. 6 ins. from the vertex on each side. Mark these joints normal to the outside curve. (The method of drawing a curve parallel to a given one which is not a circular arc, is shown at Fig. 175.)

(4) A bending moment diagram is required for representing graphically the bending moment at any point of a uniformly loaded beam, as Fig. 176. The maximum value of the bending moment occurs at the

centre of the span, and is known to be 180 tons-ins. and to vary as a parabola. Draw the curve to a scale of 1 in. = 100 tons-ins., and read off the moment of bending at a point 4 ft. from one end.

Answer, 100 tons-ins.

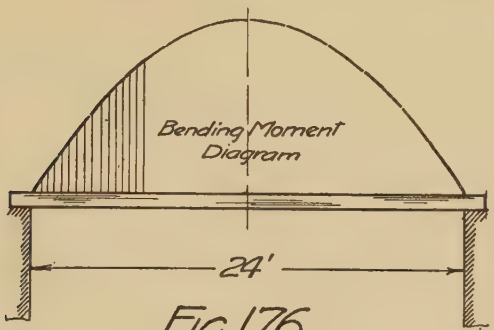


Fig. 176.

(5) Set out Fig. 174 to a scale of 1 in. = 2 ft. when the span is 10 ft. and the rise 7 ft. Choose a point Q 4 ft. from the vertex, and determine a tangent and normal at Q.

(6) The area of a parabola may be shown to equal $\frac{2}{3}$ of the enclosing rectangle. Find the area, in square feet, of the parabola you have set out in Example 5. Find a method of measuring approximately the length of the curve. *Answer*, area = $46\frac{2}{3}$ sq. ft.

CHAPTER XIII

THE SPHERE

THE sphere is another "solid of revolution" and may be described as the "solid body traced out in space by the revolution of a plane semi-circle about its diameter as axis."

The "surface" of the sphere is described by the "outline" of the semi-circle.

In Fig. 177 the semi-circle of radius R revolves about its diameter *dd* (which, prolonged, is marked AA). AA is the line or "axis" of revolution.

It is easy to see that the solid form (or surface) thus traced is the same shape at every cross section passing through the centre of the sphere, for $cd = R$ and Rc continued would serve as the axis of revolution and produce the same figure.

FIG. 177.

Further, the plane section through every part of a sphere must be a circle due to its mode of generation, the largest circle occurring at the section through C, and known as the "great circle" of the sphere.

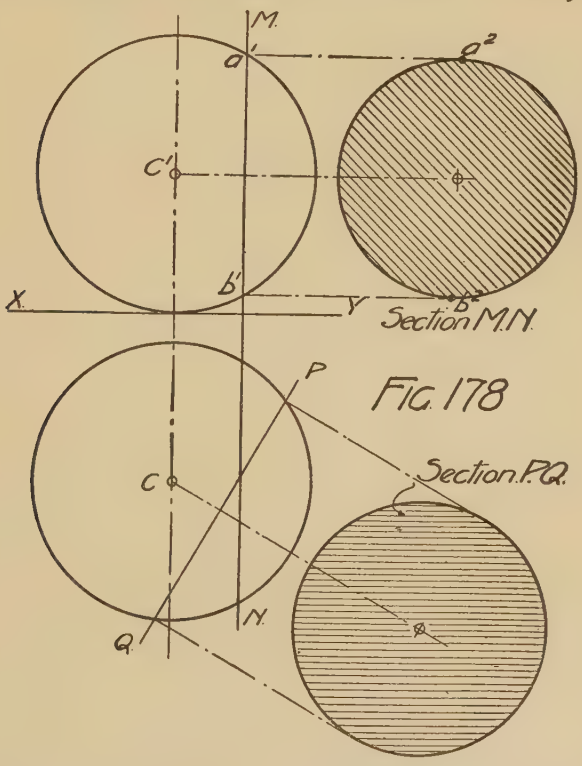
The projection of the sphere in every direction will evidently be a circle equal in diameter to the "great circle."

Let a section of the sphere be made by *any* plane, then the circle of section will have its diameter visible in the "edge view of the cutting plane." Thus, let the plane MN cut the sphere whose plan and elevation are given in Fig. 178; a^1b^1, being the edge view of the extreme points of intersection of plane and sphere, give its diameter. A new plan (or elevation) on a plane parallel to MN, and showing the true shape of the section, is constructed by projecting the diameter AB and the "centre of the circle of section." The latter always lies opposite the centre of the sphere, for, if a perpendicular be dropped from the centre of the sphere to the cutting plane, because $a^1b^1c^1$ forms

an isosceles triangle, and a^1b^1 are on the circle of section, the perpendicular from c^1 to a^1b^1 must bisect the line.

Another section is shown across the plan by the vertical plane PQ. A "new elevation" determines the true shape; the centre and diameter of the circle only, need be projected.

If these spheres be viewed when severed by planes such as MN and PQ, and the "direction of view" be other than 90° to the



plane of cut, we shall get an inclined view of the circle of section which, according to our earlier considerations of the circle, must produce an ellipse. This condition was discussed at p. 121, vol. i.

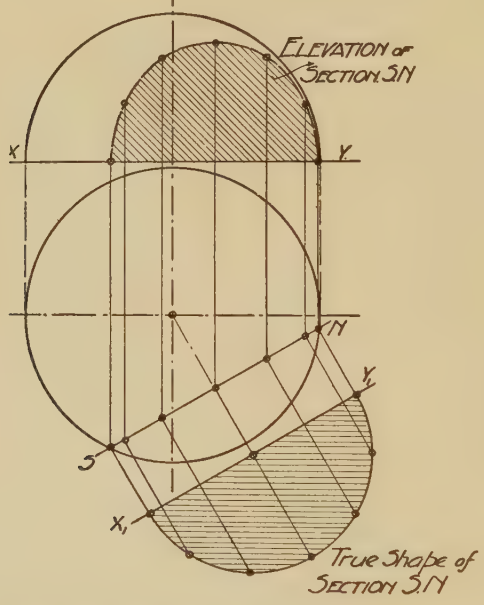
Fig. 179 is the plan and elevation of a hemisphere resting with its plane face on the ground as it would in a "dome." To become acquainted with the method of projecting the views of inclined sections we have assumed that the vertical plane SN cuts obliquely across the plan and desire an elevation of the solid when the front segment is removed.

It is desirable first to obtain an auxiliary elevation of the section made by SN. This is drawn on X_1Y_1 and is a semi-circle

with its centre opposite that of the sphere on the ground plane and diameter equal to SN. Divide this elevation of the semi-circle into a number of equal parts, and project their plans on to SN. We then have an edge view of the semi-circle and the "plans of certain points, whose heights are known."

Project an elevation of this section on the V.P. by erecting perpendiculars from each point on SN in plan and measuring

FIG. 179.



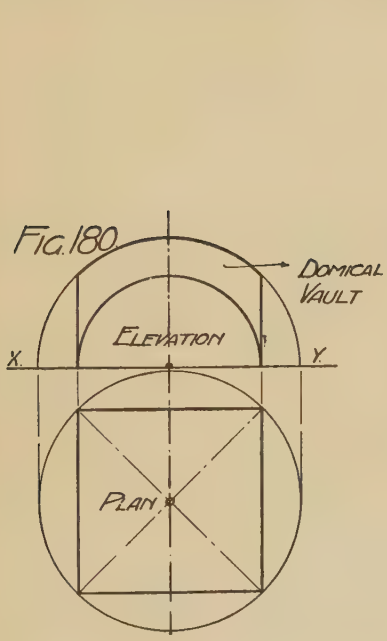
the heights from the true shape laid out in elevation at X_1Y_1. Draw the fair curve through the points so obtained, and note that the curve is a true semi-ellipse with its axis vertical.

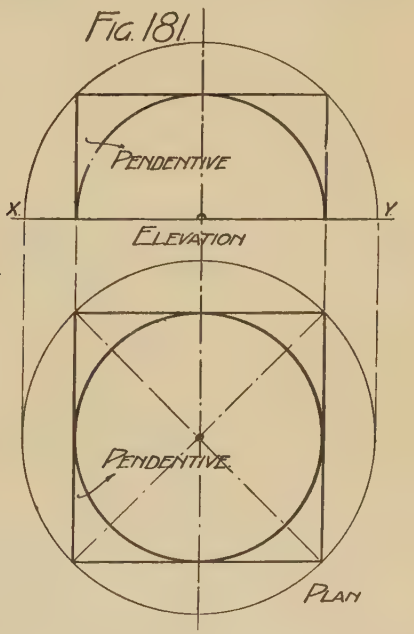
In structural problems we meet the sphere most often applied in the decorative feature known as a dome, which is a closed curved roof to an apartment, the curves of the covering usually springing from the same level all round the base. The covering arch (or arches) may spring from a square, polygonal, or circular plan, the latter being most common and architecturally the best form.

A structure raised on a circular plan may be of any convex form in cross section, *e.g.*, a semi-circle, two intersecting segments of circles, a vertical semi-ellipse, or a parabola compounded with circular arcs.

One common form (whose suitability we have not space to

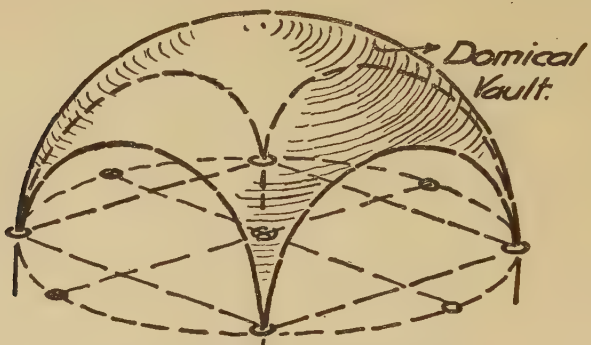
discuss here) is the hemispherical dome, of circular plan and semi-circular cross-section, therefore identical in form with the uncut solid of Fig. 179. As a dome is usually a comparatively "thin shell" of material we will think of the hemisphere as having surface only for the present. Should the dome be erected on a cylindrical wall no sectional problems are involved with it, but only constructional ones. On the other hand, should we require a dome to be erected over a square plan a conversion of form from the square to the circular must be effected, on which the true dome may rest. The problems immediately following are important for application in these cases. Typical



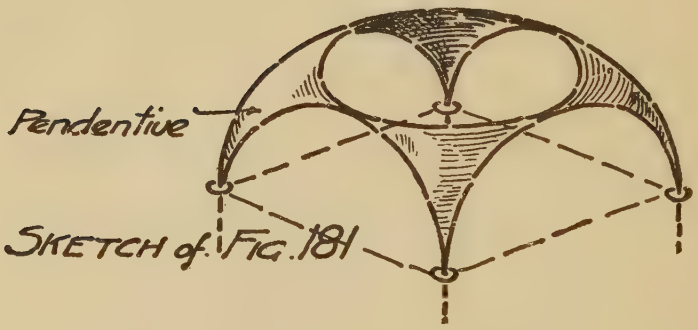


spherical sections are illustrated in Figs. 180 and 181, and the sketches attached to these.

In Fig. 180 we have a hemisphere, part of which is supposed to be utilised in covering a square plan, when the square is exactly contained within it. The four vertical walls then become vertical section planes, removing four "half-zones" and leaving the solid as shown in the sketch to Fig. 180. The outline diagram of the severed sphere is made clear in the strong lines of Fig. 180, and the resulting form of covering to the square apartment is called a "domical vault," being neither a true dome nor a true vault.



SKETCH of FIG. 180



SKETCH of FIG. 181

Sketch of Pendentives.

FIG 182.

In Fig. 181 and its accompanying sketch the top "zone" of the sphere has been removed from our result of Fig. 180, leaving a circular ring on which a true dome may be erected. This is a very simple geometrical form, though in appearance giving the idea of intricacy. Observe the spherical spandrel above each of the angles of the square. These are technically termed "pendentives" (though one cannot justify the term as applied to the construction adopted in these cases).

Fig. 182 shows a form of pendentive construction with more of the sphere remaining, the horizontal section plane being lifted, thereby reducing the size of the upper dome.

We have only concerned ourselves in the foregoing paragraphs with the geometrical conditions of the hemisphere applied to dome construction. These would be further involved with constructional difficulties which you will meet in due course, together with applications and variations of the principles in niches and vaulting.

We shall now turn our attention to the development of the sphere. Really, this is an impossibility, for no surface is truly developable except we may lay straight lines in it in "one direction" at least. Hence approximations have to be resorted to in order to produce a practically acceptable result in covering domes with wood, as a basis for lead or copper covering.

Fig. 183 shows the plan and elevation of a hemispherical dome. We require to find an *approximate method* of developing it. Consider the right-hand half of the figure.

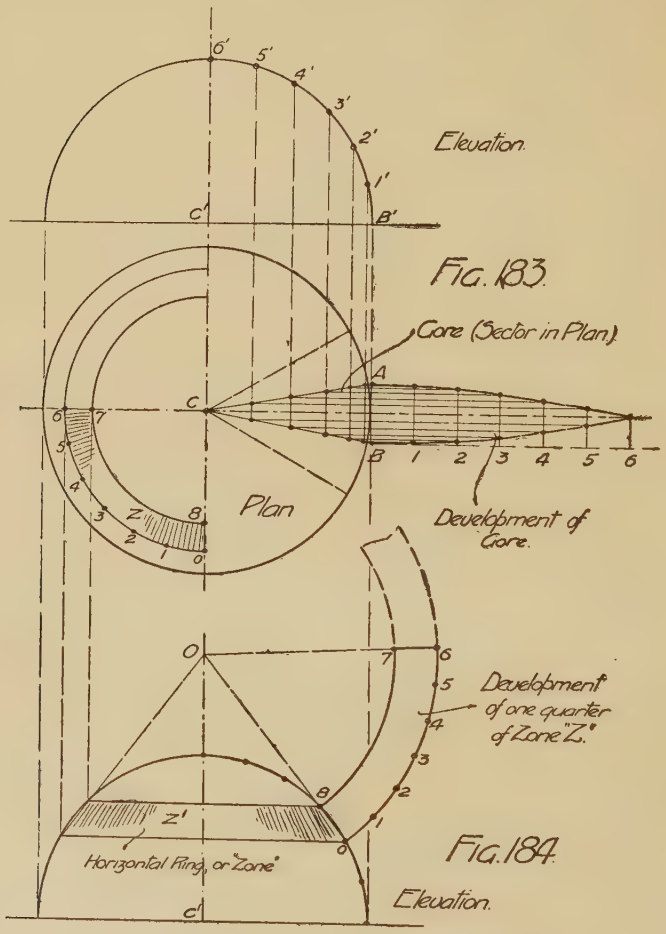
Suppose, first, that the base of the dome, instead of being a circle, is a regular polygon with a very large number of sides, and the vertical cross-section through the centre of one edge of the base remains a semi-circle. It would then be possible to develop the surfaces as in the case of Figs. 154 and 157, for each surface would be a "portion of a cylinder." Hence our approximation, which is as follows:—

Divide the base into a large number of parts such that each division is relatively very narrow as at AB in plan; join the points AB on the curve to the centre C, and then proceed to unroll this "gore" (really a sector of the circle in plan) outwards. To do this, observe that the gore ABC in plan is arranged with its centre line horizontal, so that its elevation is the quadrant 6^1B.

Divide 6^1B into equal parts, as shown in elevation; project these divisions to the plan as horizontal lines "crossing and measuring the width" of the gore.

In unrolling project the two base points on to the straight line, tangential at AB; continue the centre line through C

outwards to the right and at 90° to AB. Measure out on this line the elevation divisions $B1^1$, $1^1.2^1$, etc., to the "actual length of the curve," and mark these divisions by lines parallel to AB. Then, taking each width in plan (correspond-



ing to 1^1, 2^1, 3^1, etc., in elevation), project them to their unrolled lines in the development.

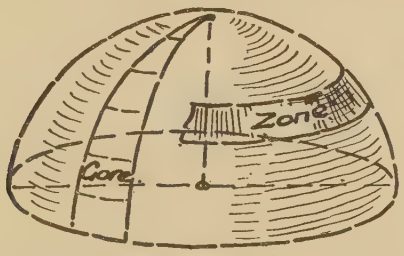
We have thus combined the "real width" with the "real length" of the approximate gore as measured round the curve at several positions, and the fair curves joining the series of developed points enclose the required area.

The development of this "gore" is not quite true, but if

the gores are sufficiently narrow the approximation is very close. In practical cut and cover work a slight extra width is allowed for bending to the curve where the gore is broadest.

The freehand sketch attached to Fig. 183 illustrates the hemisphere, and the gore marked upon it, and indicates the horizontal lines across the gore employed in its development. If you assumed this gore to be covered with paper exactly fitting it (covering the plan triangle) and pulled from the sphere by starting at the crown and drawing tightly outwards until it reaches the ground plane, you have an illustration of our process. Develop a gore, cut it out and fit over a wooden hemisphere if you are not clear on the method.

A second method of approximate development is based on the assumption that the sphere is made up of a very large number of conical surfaces forming horizontal rings in



SKETCH OF FIGS. 183 AND 184.

elevation. Such a ring (with the spherical surface retained) is called a "zone," and lies between two parallel planes. Fig. 184 is the elevation of a hemisphere (to be used with the left-hand part of the plan in Fig. 183) illustrating the method of developing by "zones." *See* sketch also.

Taking any horizontal strip, *e.g.*, Z (which must really be narrow), let it be bounded by the chord of the arc and produce this chord to the centre line or axis c^1O.

Then O is the vertex of the cone whose surface is approximately the same as that of the sphere for the zone Z. Having developed the cone previously, you are in a position to employ the principle to this example. The radius of curvature is obviously 0.8 for the inside and Oo for the outside, and the distances to be measured round the developed arc are obtained from a plan of the zone (to the left, above). This plan is marked Z, and is divided into numbered parts whose "real lengths" have been transferred and numbered correspondingly at the development for one quarter of the zone.

In preparing the boarding, the boards for one ring might

be cut of any length for suitable jointing on the framing *and of the curvature developed for that ring.*

You should develop two other zones for practice in applying the principle, and you will observe that the various segments may be sufficiently developed on the same side of the sphere without overlapping.

Many further problems concerning the sphere will need to be considered in its application to building and architecture, but our space will not allow of further consideration. We might briefly note in passing that "niches" are semi-domes and have the same surface properties, and that "vaults" are usually portions of intersecting cylinders cutting each other in various ways, and, like a dome, forming a covering to an apartment. As "form" is always involved in "construction," your acquaintance with the geometrical conditions should make future study of these architectural features both easy and interesting.

Before closing our chapter on the sphere we must briefly consider its "area" and "volume."

The surface area of a sphere is well known to be equal to that of the "enclosing hollow cylinder," hence if the radius of the sphere be r the diameter is $2r$, and therefore the length of the enclosing cylinder is $2r$.

$$\begin{aligned}\text{Area of cylinder } (A_{cy}) &= \text{circumference} \times \text{length} \\ &= 2\pi r \times l \\ &= 2\pi r \times 2r\end{aligned}$$

$$\text{and area of sphere } (A_s) = 4\pi r^2.$$

It is therefore easy to remember the "statement" and obtain the formula when required.

The volume of a sphere cannot be so easily stated for memorising, but may be proved to be—

$$\text{Volume of sphere } (V_s) = \frac{4}{3}\pi r^3$$

Then as $\pi = 3.142$ and $r = \frac{d}{2}$ (diameter)

$$V \text{ also} = .5236d^3.$$

It is well to remember that the cubic contents of a sphere are a little in excess of "half the volume of the enclosing cube." This forms an excellent practical check on calculations, and should always be applied to test the correctness of a result obtained by the "radius formula."

We may now give examples involving the work of this chapter.

EXAMPLES ON THE "SPHERE IN CONSTRUCTION"

(1) Draw the projections of a full sphere of 4 ft. diameter, scale 1 in. = 1 ft. Across the plan place a vertical section plane at 30° to the XY, $\frac{1}{2}$ in. clear of the "centre." Obtain an elevation of the sphere with the fore part removed, as in the hemisphere of Fig. 179.

(2) Examine Fig. 180 again. Suppose the diameter of the plan is 8 ft., and the cutting planes form a regular hexagon on the plan instead of a square, with the hexagon lying within the great circle, obtain an elevation of the "remainder of the sphere."

(3) In Problem 2 let the upper portion of the sphere be removed by a horizontal plane touching the circles of section made by the vertical planes, so that its condition is similar to Fig. 181. Now complete the plan and elevation of the sphere, showing its six pendentives. All the above is to be worked as a line diagram only.

(4) Take a hemisphere of 6 ft. diameter drawn to a suitable scale. Let this represent the exterior of a dome. Develop the shape of a gore 6 ins. wide at the base, which could be cut and bent approximately to fit the sphere. The edges of the gore are in vertical section planes, passing through the centre of the sphere.

(5) How many gores like your development of question 4 would it take to cover the whole surface? State the area of the surface of the hemisphere. *Answer*, 38 gores.

(6) A quarter spherical niche is to be covered with boards in horizontal rings. The radius of the sphere is 3 ft., and the rings are to be 5 ins. parallel width (or as near as possible). Choosing the two zones nearest the base and the two at the top, develop them by the conical approximation, as per Fig. 184.

(7) Find the area of a hemispherical dome which is to be covered with lead. Its external diameter is 15 ft. What would be the cost of covering with 6 lbs. lead at 21s. 4d. per cwt., if 10 per cent. of the surface covered be allowed for laps and waste?

Answer, Area (nett) = 353.5 ft. superficial.
Weight (gross) = 20 cwt., 3 qr., 8.8 lbs.
Cost, £22, 4s. 4d.

(8) Calculate the volume of a sphere whose diameter is 11 ft.
Answer, 697 cu. ft.

(9) A dome consists of a hollow hemisphere, external diameter 12 ft., shell 9 ins. thick. Obtain—
(*a*) Its volume of stonework.
(*b*) Its outer surface area.
(*c*) Its inner surface area.

Approximate answer, Volume = 149 cu. ft.
Outer area = 226 sq. ft.
Internal area = 173 sq. ft.

CHAPTER XIV

MEASUREMENT OF IRREGULAR AREAS—GEOMETRICAL REDUCTION

FROM vol. i., chap. ii., a considerable amount of knowledge will have been gathered relating to the triangle, and the properties of the triangle as there detailed must be remembered and applied as required.

The foregoing work has dealt with the following points:—

(*a*) Measurement of angles.
(*b*) Particulars necessary for the construction.
(*c*) The right-angled triangle and the relation of areas of squares constructed on its sides.
(*d*) Area of the triangle and of figures compounded of triangles.
(*e*) Properties of similar triangles and their application to the division of scales.
(*f*) Use of trigonometrical ratios.

We are now able to develop our knowledge in the further application of the properties of the triangle to the solution of problems.

REDUCTION OF AREAS

In computing areas, for instance, we can often save much tedious calculation "in cases where a drawing of a figure is necessarily provided for some ancillary purpose," not *primarily* for calculation.

Complicated figures may be reduced to simple ones in a brief period, and with sufficient accuracy by converting the form in stages, from the many sided or irregularly curved figure into a triangle or a square.

To introduce the method we must refer to the properties of similar triangles at p. 10, vol. i. Fig. 185 will serve for convenient reference. It consists of two similar triangles superposed—placed one over the other so that two pairs of

sides of the triangles coincide and meet at a common point B. Thus the angle B is the same in both triangles. Also, because DE is parallel to AC, the angles D and E are equal to A and C respectively. Two triangles are "similar" when they "differ in size" but have equal corresponding angles, and when this condition exists the pairs of similar sides have the same ratio to each other. We might say that BD is contained in BA as many times as BE is contained in BC. It therefore follows that we may write $\frac{BA}{BD}=\frac{BC}{BE}$, and because the ratio is still true for the remaining pairs of sides we might have continued $\frac{BA}{BD}=\frac{BC}{BE}=\frac{AC}{DE}$. This property enables us to solve problems either by drawing the triangles to scale, or if given three quantities (lengths) in any of the above pairs of ratios we may "calculate the fourth."

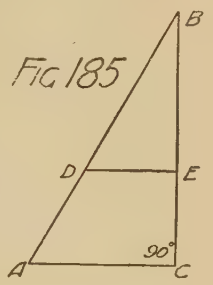


Fig 185

You will only realise the value of the above by following them up further. Thus, if $\frac{BA}{BD}=\frac{BC}{BE}$, by your previous mathematical training you know that by cross multiplication and division we should get (multiplying both sides by BD)—

$$BD\times\frac{BA}{BD}=BD\times\frac{BC}{BE}$$

$$\therefore\ BA=\frac{BD\times BC}{BE}.$$

Then dividing both sides of our result by BC, we have—

$$\frac{BA}{BC}=\frac{BD\times BC}{BE\times BC}$$

$$\therefore\ \frac{BA}{BC}=\frac{BD}{BE}$$

and similarly we can show that the pairs of corresponding sides in similar triangles always bear similar ratio to each other, in whatever way they be compared. The following should be carefully studied:—

In Fig. 185 $\quad\frac{BE}{BD}=\frac{BC}{BA}=\frac{DE}{AC}$

Also $\quad\frac{EC}{DA}=\frac{BE}{BD}=\frac{BC}{BA}$

where EC and DA are "differences" between pairs of similar sides.

As a rule, our calculations and problems are not complicated, and if you are clear that these ratios apply to triangles of all kinds *when similar*, and can pick out the *similar sides* or *parts of sides* you will have no difficulty in gripping the problems.

Recall the fact that for right-angled triangles having base angles from 1° to 89° the constant ratios between pairs of similar sides are calculated and tabulated for speedy reference and "named" for distinction. These have been called "trigonometrical ratios" (measures of the relationship between the three sides of triangles under stated conditions). We shall return to them later.

For the present let us use the more direct application of the similar triangle.

Assume a rectangle whose sides are AD and DC in length, and a square is required having the same area as the rectangle.

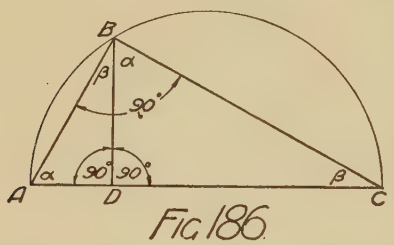


Fig 186

Let the side of this square be called BD, then, because the areas are to be equal, we have $AD \times DC = BD^2$.

Set out AD and DC in a straight line as at Fig. 186. Erect a semi-circle on AC. You learnt in vol. i. that if any point on the semi-circle be joined to A and C the angle between the lines at such point would be 90°. (This was stated: the angle in a semi-circle is constant and equals 90°). Erect a perpendicular at D intersecting the semi-circle at B. Join B to A and C, then angle ABC is 90°, and the angle on each side of DB is 90°. If the angle at $A = \alpha$, and that on the left of the perpendicular at $B = \beta$, from previous work we know that the sum of the angles A, B and D = 180°. Therefore $\alpha + \beta + 90° = 180°$, and that consequently $\alpha + \beta = 90°$.

From this it is clear that the angle on the *right* of the perpendicular at $B = \alpha$ for the full angle at $B = 90°$ and $\alpha + \beta = 90°$.

Further, as the triangle BDC is a right-angled triangle angle C must also equal β.

Hence the triangles ABD and BCD are "similar."

To make sure the conditions are understood we have separated and redrawn them at Fig. 187, while maintaining the same lettering. From these individual triangles we note that—

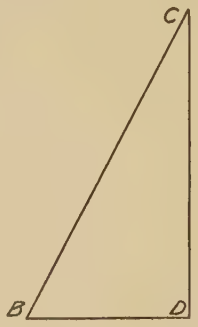
$$\frac{AD}{BD}\ (\text{small triangle}) = \frac{BD}{CD}\ (\text{large triangle}).$$

Observe that BD is the same in both triangles, therefore

by transposition $AD \times CD = BD^2$, by which we establish the fact that the square on BD is equal in area to the rectangle whose sides are AD and CD.

This line BD is called the *mean proportional between AD and DC*, for the pairs of ratios may be written in the usual form of a proportion, thus—

$$AD : BD : : BD : DC.$$



The middle terms are alike. You probably know the rule which states that "the product of the *means* equals the product of the extremes" in any proportion. The "means" are the *middle* terms, the "extremes" are the *end* terms; we therefore obtain by multiplication of these the same result as before, viz.:—

$$AD \times DC = BD^2.$$

FIG 187.

The term "mean proportional" should now be clear.

Problem.—To find the mean proportional between two quantities, or (which is the same thing) to find the side of a square having the same area as a rectangle whose sides are known.

Let the sides of the rectangle (or the two quantities) be represented by EF and FG (Fig. 188). Set these values in

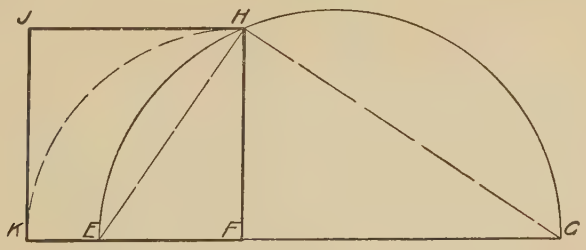


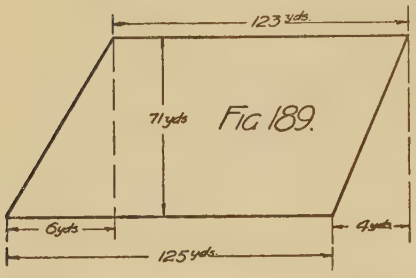
FIG 188

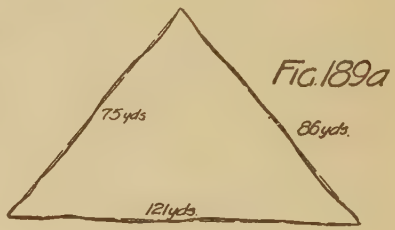
a straight line, describe a semi-circle on EG, and erect a perpendicular at F to meet the semi-circle; then FH is the "side of the required square," or *the mean proportional.* The square having the same area as the rectangle EF, FG, has been constructed on FH to the left. (The rectangle is not shown as a bounded area, in order to avoid confusion of lines.)

EXAMPLES ON "PROPORTIONALS"

(1) A plot of ground is rectangular, and measures 18 yds. × 72 yds. By the mean proportional method find the length of side of a square field having the same area. *Answer*, 36 yds.

(2) Another plot of ground is shaped as per Fig. 189 (not to scale). Convert this geometrically into a square of equal area. What is the length of its side? *Answer*, 93.83 yds.





(3) Convert the roughly triangular plot of ground (Fig. 189*a*) into a square by geometrical construction. Let the sides be approximately $a = 75$ yds., $b = 121$ yds., and $c = 86$ yds. Notice how the dotted lines approximate to the irregular ones in regard to the amount of ground they enclose. *Answer*, Side of square = 56.56 yds.

Check your "resulting area" by the following formula :—

$$\text{Area} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

where a, b, and c are the lengths of the sides of the triangle, and s = half the sum of these sides, viz. :—

$$s = \frac{a+b+c}{2}.$$

If you have learnt to use logarithms you will find the above an easy method of computing the area of a triangle *when given the lengths of its sides*. It is perfectly accurate, but should not be employed when the "height" of the triangle and the "length" of the base are known. *Answer*, $s = 141$.

Area = 3199 sq. yds.

(4) Convert the result of question 3 into land measure.

Answer, 2 roods 25 poles $22\frac{3}{4}$ sq. yds.

The preceding examples have required the conversion of rectangles, parallelograms, and triangles to the form of a square. In each case the original figure was very simple, and the problem almost self-evident. We must now consider cases where the method is more complicated.

In the earlier work you have probably established a theorem which states: "Triangles on the same base, and having the same perpendicular height, are *equal in area.*" By using this theorem we can reduce irregular polygons to the triangular form very rapidly, and then apply the triangle rule to measure the area (or, if desirable, reduce to a square).

In the application of the theorem we must notice that the perpendicular heights of two triangles are the same when "contained between two parallel lines," which occurs when the bases are on *one* such line, and the vertices in the other.

Consider Fig. 190. A trapezium, ABCD, is to be reduced to a triangle, and then to a rectangle of equivalent area. Join AC, cutting off the triangle ABC. Consider AC as the base. From B draw parallel to AC to cut AD produced at E. Join EC. Now AC is the common base of the two triangles

FIG 190

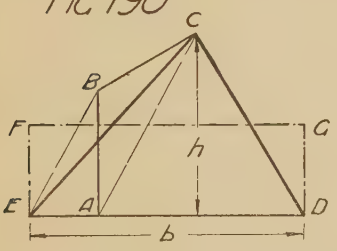
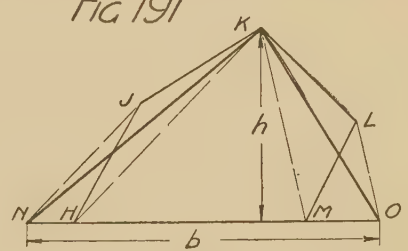


FIG 191



ABC and AEC, and both have their vertices on the parallel EB, thus giving them the same perpendicular height. Hence the triangles are equal in area. Let ECD take the place of the original figure ABCD. It has the same area, for the remainder of the original figure has not been altered in any way.

Observe that our object in this process has been to replace one triangle by another, so that the original figure has had its number of sides reduced by "one," and that the substituted triangle enabling this reduction to be made had *one of its sides lying in the base produced.*

The rectangle having the same area as ECD lies on the same base, and is half the height of the triangle. It is evident, then, that the rectangle EFGD is equal in area to ECD for in each case area $= \frac{bh}{2}$.

Fig. 191 shows the same principle of reduction applied to an irregular pentagon, HJKLM.

HJK is cut off and replaced by HNK, NH lying in the original base. KLM is intercepted and substituted by KOM,

therefore NKO is an equivalent triangle whose area $= \frac{bh}{2}$ as before.

Let us now solve a more irregular area such as Fig. 192. Its curved boundary is divided into segments so that each is approximately a straight line. DEB is removed and

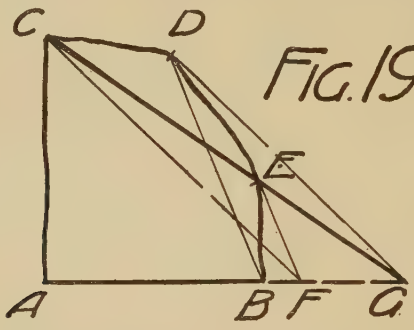


FIG. 192.

replaced by DBF, then CDF removed and replaced by CGF. (Observe that the line DF has not been drawn. It helps to avoid confusion of lines by omitting these unnecessary ones.) Our operations result in a fair approximation to the area by measuring the area of the triangle ACG.

Fig. 193 carries the principle a step further. The figure contains what is known as a re-entering angle, at L. The triangle

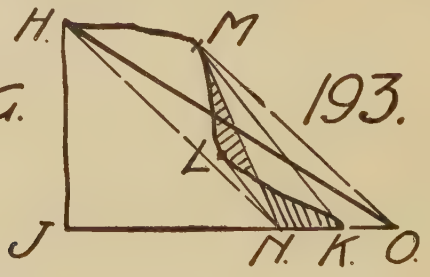
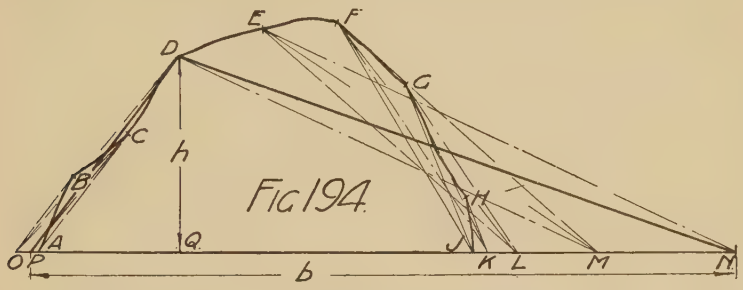


FIG. 193.

MLK is enclosed by MK. . . . LN is parallel to MK. Therefore, area MKN = area MLK. As the part of *both* these triangles lying outside the original figure, and on the base MK is *common*, and the *whole triangles were equal* in area, the differences (shown shaded) must also be equal, and as one shaded area has been included and the other excluded by the new boundary MN, the resulting figure JHMN equals the original one.

If HMN be now replaced by HON we have the final triangle HOJ of *equivalent area.*

Fig. 194 further develops this application, which is well known to land surveyors. The area of the diagram is required; it is very irregular due to the zigzag boundary. We have a choice of methods for measuring it, viz.:—(*a*) the *mid-ordinate* (or some similar) method; (*b*) the reduction to a triangle of equal area.



We shall follow up the "method of reduction" first. Divide the boundary into short and approximately straight lengths as AB, BC, CD, etc., to HJ, then proceed on the principles of Figs. 192 and 193 as follows:—

On the right—

1. Remove GHJ and substitute GKJ.
2. „ FGK „ FLK.
3. „ EFL „ EML.
4. „ DEM „ DNM.

Let D be the vertex of the final triangle.

On the left—

1. Remove CBA and substitute COA.
2. „ DCO „ DOP.

Then PDN is the triangle of equal area.

$$\text{If } PN = b \text{ and } DQ = h$$
$$\text{then area of plot} = \frac{bh}{2}.$$

Should the plot of ground be "long" and "narrow" the method of reduction is not very successful because the flat slopes occurring in the process are liable to errors in drawing.

In *any* case the mid-ordinate method will serve equally well, and is particularly suited to long and narrow figures.

Let Fig. 194*a* be roughly similar to Fig. 194. Divide the base into any convenient number of equal parts, say 8, and erect "ordinates" at 90° to the base which are lettered in the diagram BB_1, CC_1, etc. At the centre of each vertical strip of the divided area erect a "mid-ordinate." Let the

equal divisions of the base (width of a strip) be s units across.

Now consider *one* strip, say CDD_1C_1. Assume CD to be a straight line. Its "mean" or "average" height would be the length of the mid-ordinate p and its area, $p \times s$.

If we agreed to assume that all the boundaries AB, BC, CD, etc., were straight, and that the "mean height" should be

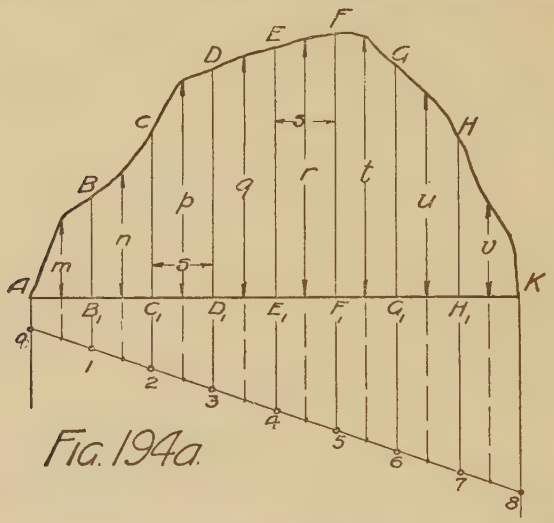


Fig. 194a.

represented by the mid-ordinate of the strip, we have only to sum up the "mean lengths" of the strips and multiply by the common width s to obtain the total area.

Now you can see that, because the boundary is a curved one —instead of being straight as assumed—the mid-ordinate is not the "correct mean length," but is only an approximation. In this particular strip CDD_1C_1 we have measured the ordinate *to the curve*, and it is thus *slightly in excess*.

Observe, however, that when the boundary is tortuous we shall sometimes be measuring in *excess* and at other times *under* the correct dimension. Therefore the ultimate error is somewhat reduced. Again, if the strips are taken sufficiently narrow, the errors become very small.

In summing up the lengths of the strips to obtain the total and multiplying by the width s, we might write our operation thus:—

$$\text{Area} = s\,(m+n+p+q+r+s+t+u).$$

To make the calculation as easy and accurate as possible you should choose s some easily measured

distance, or obtain it by writing the full length of the base $b \div$ the number of strips ; then, in this case

$$\text{Area} = \frac{b}{8}(m + n + p, \text{ etc.}, + u).$$

Examine the construction below the base of the figure, 194*a*. The divisions have been obtained by laying the scale rule between two perpendiculars dropped from the ends of the base, so that the required number of divisions on the rule (of any convenient and equal length) fitted across diagonally. *See* vol. i., p. 9. You may also employ the method described on pp. 10 and 11.

The same mode of measurement may be applied to any irregular figure, whether its boundary has a straight side or not. If no straight line exists place one outside the figure approximately in the longest direction. Draw ordinates at 90° to this across the figure, and measure *the mid-ordinates of the strips between the boundaries.*

EXAMPLES ON IRREGULAR AREAS

(1) Set out "any irregular figure" representing a plot of ground, as in Fig. 194. Use (*a*) the method of geometrical reduction, (*b*) the mid-ordinate method of measurement, and determine the area of the plot. Your results should not vary more than 2 per cent.—good work not more than 1 per cent.

(2) A foundation trench, 200 ft. long and 11 ft. deep at the higher end of the ground, has to be dug 4 ft. wide. The longitudinal section is

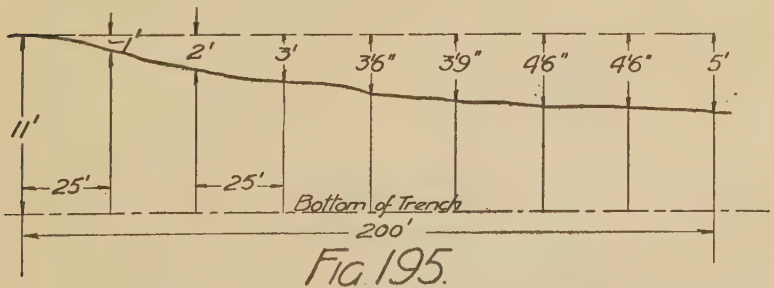


Fig 195.

given in Fig. 195. Find, by the mid-ordinate method, the amount of earth removed. Answer in cubic yards. If it increases in bulk 8 per cent. when dug, how much space has to be provided for tipping it?

NOTE.—The *mid*-ordinates are not given here, but you can take each as the average between the ordinates supplied.

Answer, 206.48 cu. yds., and 223 cu. yds.

(3) In vol. i. you have been informed that the area of an ellipse is expressed by $\frac{\pi}{4}Dd$ (or $.785Dd$) where D is the major axis and d the minor axis (*see* vol. i., p. 126). Set out an ellipse 16′ × 10′ to scale. Determine its area by the mid-ordinate rule, and see that it agrees approximately with the calculated result.

Answer, area = 125.6 sq. ft.

(4) In Fig. 196 is given the plan of a piece of ground, with the surveyed lengths of sides to the main triangle, and the offsets made at intervals

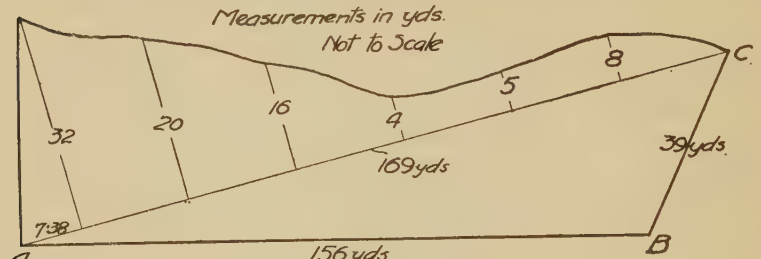


FIG. 196.

along the hypotenuse of the triangle, to determine the irregular boundary. By the mid-ordinate method, or any combination or variation of it, measure up the area of the plot of ground illustrated in the figure. *Answer*, 4897 sq. yds.

CHAPTER XV

ELEMENTS OF TRIGONOMETRY

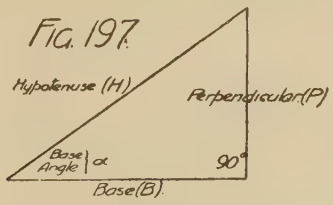
In vol. i., pp. 12-14, a brief explanation of the three common ratios between the pairs of sides of a right-angled triangle was given. We shall here provide an opportunity for their use in a series of very elementary applications.

The ratios were referred to as the common trigonometrical ratios and named as follows:—

Referring to Fig. 197 and to the base angle α, it was stated that—

$$\left.\begin{aligned} \frac{P}{H} &= \text{sine } \alpha\text{—abbreviated to sin } \alpha \\ \frac{B}{H} &= \text{cosine } \alpha \quad \text{„} \quad \cos \alpha \\ \frac{P}{B} &= \text{tangent } \alpha \quad \text{„} \quad \tan \alpha \end{aligned}\right\} \text{A.}$$

According to the variation in the size of the angle, so we get varying values of these ratios. Tabulated values for angles varying by 1° are given at end of the book.



Now observe that because these statements refer to the equality of two ratios they might be written thus:—

$$\frac{P}{H} = \frac{\sin \alpha}{1}$$

which is equivalent to saying that when H = 1, P = "the value assigned to the sine of the angle in the tables." Don't look upon these tables of ratios as something mysterious: they are perfectly simple expressions of the relative lengths

of the sides for a given angle, and always referred to the "right-angled triangle."

Suppose $a = 30°$. From our tables—

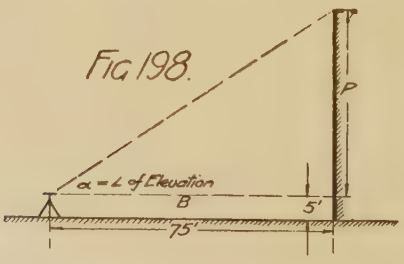
$$\sin 30° = .5 \quad (a)$$
$$\cos 30° = .866 \quad (b)$$
$$\tan 30° = .577 \quad (c)$$

This means that in (*a*) the perpendicular = .5 when the hypotenuse = 1 ; in (*b*) the base = .866 when the hypotenuse = 1 ; and in (*c*) the perpendicular = .577 when the base = 1, in whatever units you like to employ. It thus follows that from the tabulated ratios we may set out any angle by plotting the values along the proper sides of the triangle while maintaining the right angle between the perpendicular and the base.

We may also make calculations from any of the three simple equations at A to obtain any single unknown quantity —as a "side" or the "base angle"—if we choose the correct ratio containing the unknown symbol.

We shall work through a few examples to show the usefulness of the method, and in doing so express our recognition of the fact that these examples are of far greater value to the "land" and "quantity" surveyor than to the architect and builder. On the other hand, a real knowledge of even the elements of trigonometry may be made to save much time in checking measurements.

Example 1.—We desire to know the height of an object which is inaccessible, such as a church tower. Standing at a distance of 75 ft. from its base on level ground it was found that the angle of elevation (inclination upwards from the



horizontal) was 42 ins. when the instrument for measuring the inclination was 5 ft. above the level of the tower base. Find the height. Fig. 198 shows the conditions; the right-angled triangle shown has a known "base length" and a

known "base angle," and as we require the perpendicular height the "tangent" expression is suitable for use.

$\alpha = 42°$
$\tan 42° = .9004$
$B = 75$ ft.

We know that $\frac{P}{B} = \tan \alpha$

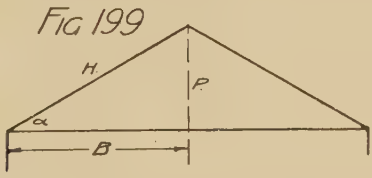
By transposition $P = B \tan \alpha$

and inserting values, $P = 75 \times .9004$

$= 67.53$ ft.

But, as the instrument was 5 ft. above the base of the tower its total height $= 67.53 + 5 = \underline{72.53 \text{ ft.}}$

Example 2.—A roof has an inclination of 35°. Its span is 42 ft. Find the length of its rafter without setting it out and measuring. Fig. 199 illustrates the roof. It is evident that



the triangle for solution is that on the "half" span. B is known, α is known, and we require H which indicates the use of the expression containing the cosine, viz.:—

$B = 21$ ft. (half span)
$\alpha = 35°$
$\cos \alpha = .8192.$

$\frac{B}{H} = \cos \alpha$

By transposition $H = \frac{B}{\cos \alpha}$

and inserting values, $H = \frac{21}{.8192}$

$= \underline{25.6 \text{ ft.}}$

The extreme length of the rafter for actually covering 21 ft. of plan at a slope of 35° is therefore 25.6 ft.

Example 3.—A V roof is pitched at 40° to the horizontal. Its rise measures 7 ft.; what length is the rafter, and what span does it cover? Fig. 199 will again serve.

(*a*) Find length of rafter. If we know the angle α and

the rise P, and require H, the expression of the sine contains all three.

$$\text{Thus } \frac{P}{H} = \sin a \qquad \begin{matrix} a = 40^\circ \\ \sin 40^\circ = .6428 \\ P = 7 \text{ ft.} \end{matrix}$$

$$\text{By transposition } \frac{P}{\sin a} = H$$

$$\text{Reversing and inserting valves, } H = \frac{7}{.6428}$$

$$= \underline{10.89 \text{ ft.}}$$

Length of rafter is 10.89 ft.

(*b*) Find span covered. We require the base B, and have given a and P, hence we may use the expression containing the tangent.

$$\text{Thus } \frac{P}{B} = \tan a \qquad \begin{matrix} a = 40^\circ \\ \tan 40^\circ = .8391 \\ P = 7 \text{ ft.} \end{matrix}$$

$$\text{By transposition } B = \frac{P}{\tan a}$$

$$\text{and inserting values, } B = \frac{7}{.8391}$$

$$= 8.34 \text{ ft.}$$

But B is half the span,

$$\therefore \text{ span covered} = \underline{16.68 \text{ ft.}}$$

Example 4.—A span of 28 ft. is to be covered by a V roof (Fig. 199), the rafters supplied being $15\frac{1}{2}$ ft. long. What pitch may be adopted to utilise the full length of the rafters?

We know span (therefore base B), and the hypotenuse H, and require the angle a. The expression of the cosine contains these values, therefore—

$$\frac{B}{H} = \cos a \qquad \begin{matrix} B = 14 \text{ ft.} \\ H = 15\frac{1}{2} \text{ ft.} \end{matrix}$$

$$\therefore \cos a = \frac{14}{15.5}$$

$$= .9032.$$

Find the angle whose cosine is nearest to this value. It is 25°, therefore the roof pitch must not exceed 25° to use the rafters stated.

The examples we have worked show how the ratios may be employed for solving right-angled triangles. We do not suggest that you are going to employ these where a drawing is supplied or another method is more convenient or economical.

Our chief concern is that they pave the way to more difficult uses of the principles of trigonometry which must be met with later by those sections of building trades students whose professions demand it.

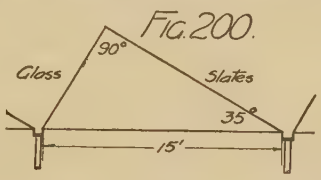
The following examples are intended to give practice in the use of trigonometrical ratios, and of the properties of similar triangles. In many cases the similarity of triangles helps one to a rapid conclusion if he fully realises the conditions of a problem. Try, therefore, to become conversant with these conditions as arising in the following examples.

EXAMPLES ON THE USE OF TRIGONOMETRICAL RATIOS, ETC.

(1) A collar beam V roof of equal slopes and 45° pitch has a span of 25 ft., and the rafters rest 10 ins. over the bearing. The "collar" is placed with its under surface $\frac{2}{3}$ of the rise below the ridge. What length of collar is required, neglecting thickness of material?

Answer, 16 ft.

(2) Section through a series of shed roofs is shown at Fig. 200. One bay is fully detailed and has a covering partially of slates and partially



of glass. Calculate the area of "slating" and of "glazing." Roof is 120 ft. long.

Answer, Area of slating = 163.84 sq. yds.
„ glazing = 1032.48 sq. ft.

Observe that the slating is usually "billed" in square yards and the glazing and sashes in square feet.

(3) Suppose that in Fig. 200 the span was 17 ft., the slate pitch 30°, and the ridge angle 90°, find the length of the rafters and glazing by any method you please. *Answer*, Length of rafter = 14.72 ft.
„ glazing = 8.5 ft.

(4) Obtain the height of a building whose angle of elevation to the highest point is 36° from a point 120 ft. distant at a level of 3 ft. 6 ins. above the base of the building. *Answer*, 90.68 ft.

(5) If a common V roof has a slope of 45°, what is the relation of the length of rafter to the half span? Let the span be 45 ft. What length of rafter is exposed?

Answer, Relation = $\sqrt{2}$: 1 : : length of rafter : half span.
Length of rafter = 31.81 ft.

(6) The plan of a hip rafter is $\sqrt{2} \times \frac{\text{span}}{2}$. Show by a plan that this is true for a square-hipped end, when the pitch of the "return" is the same as the main roof pitch. Let the span be 28 ft. and the rise 7 ft. Calculate the length of the hip. Find also the length of a common rafter.

Answer, Plan of hip = 19.8 ft.
True length of hip = 21 ft.
Length of common rafter = 15.65 ft.

(7) A bridge is to be thrown across a cutting at a height of 15 ft. from its base. The base is level, 20 ft. wide, with banks sloping at 45°. Allowing 4 ft. of bearing at each end of the bridge, find its length by calculation. *Answer*, 58 ft.

(8) Suppose the banks rise 2 vertical in 3 horizontal, what will be the new length required with the same bearings? *Answer*, 73 ft.

(9) Calculate the length of the common rafter in the spire roof of Fig. 136. *Answer*, 30.3 ft. or 30 ft. 3⅝ ins.

(10) A surveyor wishing to know the distance between two points on opposite sides of a railway cutting set up his theodolite at a point A and directed it towards the object B on the opposite side. He then turned the instrument through an angle of 90° alongside the cutting, set a pole in this line at C, and measured the distance AC along it to be 73 yds. Moving the instrument to C he directed it towards the object B, turned it into the line CA, and noted that the angle between CA and CB = 51°. Find by calculation and check by plotting the distance between points A and B across the cutting. *Answer*, 90.15 yds.

Areas of Inclined Planes by Trigonometry

We shall now show how the area of a sloping surface may be directly obtained from its plan when the angle of inclination of the surface is known.

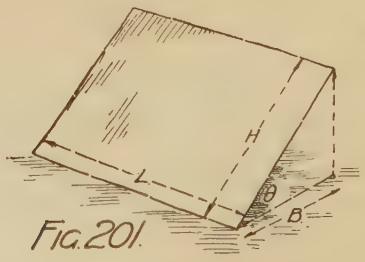


Fig. 201.

Let the slope of an inclined surface be the angle θ, Fig. 201. From an end view of the plane we can see the relationship of the length of a line of steepest slope (H) to its plan (B). The

area of the "plan" of the plane would therefore $= L \times B$. But the *real* area would be $L \times H$, as clear from the figure.

Now if $\frac{B}{H} = \cos \theta$, given B we can find H.

Let us suppose the plane to be a roof surface; given its "plan" and "angle of slope" we may calculate its real area, for,

$$\text{since } \frac{B}{H} = \cos \theta$$

$$H = \frac{B}{\cos \theta}$$

$$\text{Now the "real" area} = L \times H$$

$$\text{and by substitution for H, real area} = L \times \frac{B}{\cos \theta}$$

$$\text{But } L \times B = \text{plan area}$$

$$\therefore \text{ real area} = \frac{\text{plan area}}{\cos \theta}$$

and we have a convenient rule for application to all such cases. Given the plan of any uniformly sloping surface, no matter what the shape of the plan may be, its area equals

$$\frac{\text{plan area}}{\cos \text{ angle of slope}}.$$

When a roof, however broken, has a "constant" slope, its area is easily found, for the whole plan may be first measured and then divided by cos θ at one operation.

Take Fig. 139 as an example. Let its slope $(\theta) = 30°$ at every part of the covering; from the tables cos $30° = .866$.

$$\begin{aligned} \text{Plan area} &= 20 \times 10 + 4 \times 15 + 60 \times 22 + 2 \times 18 \\ &= 200 + 60 + 1320 + 36 \\ &= \underline{1616 \text{ sq. ft.}} \end{aligned}$$

$$\begin{aligned} \text{and "roof area"} &= \frac{\text{plan area}}{\cos 30°} \\ &= \frac{1616}{.866} \\ &= \underline{1866 \text{ sq. ft.}} \text{ or } \underline{207\tfrac{1}{3} \text{ sq. yds.}} \end{aligned}$$

Roof areas are sometimes expressed in squares of 100 ft. superficial, therefore the area $= \underline{18.66 \text{ squares.}}$

The same principle applies *no matter what the plan shape is*, so long as the correct slope to the H.P. is known. For instance, let a chimney stack measuring 3′ 9″ × 2′ 3″ pass

through a roof whose inclination to the horizontal is 45°. What is the area of deduction for the slating considered nett?

$$\text{Area of hole in roof} = \frac{\text{plan area of chimney}}{\cos 45^\circ}$$

$$\text{Plan area} = 3\tfrac{3}{4} \times 2\tfrac{1}{4} = 8.44 \text{ sq. ft. approximately}$$

$$\text{and } \cos 45^\circ = \frac{1}{\sqrt{2}} = .7071$$

$$\therefore \text{ Area for deduction} = \frac{8.44}{.7071} = \underline{11.93 \text{ sq. ft.}}$$

Here is another example.

A circular chimney stack was enclosed in the addition to a factory, the structure being built around it, leaving the chimney protruding through the roof. The chimney had an external diameter of 8 ft. at the roof level, and the inclination was specified as $\frac{1}{3}$ pitch. Find the nett area of deduction in the sarking boards with which the roof was covered, and also the number of feet run of cutting and waste in trimming the boards round the stack. (You may check your results partially by drawing and measurement.)

As the rise is $\frac{1}{3}$ the span, we have the perpendicular (P) = $\frac{2}{3}$ half span. Hence $\tan \theta = \frac{P}{B} = .6666$, and the angle in degrees whose tangent is nearest to this, is 34° ($\tan = .6745$).

$$\text{The plan area} = .785d^2 \qquad d = 8 \text{ ft.}$$

$$\therefore \quad \text{,,} \quad = .785 \times 64 \qquad \cos 34^\circ = .8290$$

$$\therefore \text{ Area of hole} = \frac{.785 \times 64}{.829}$$

$$= \underline{60.6 \text{ sq. ft.}}$$

We might have found this by another method.

If the rise is 2 ft. for each 3 ft. of base, the total rise at the intersection of the chimney from side to side = $\frac{2}{3} \times 8' = \frac{16}{3}'$. By the property of the squares on the sides of a right-angled triangle, the major axis of the ellipse (along the line of steepest slope) is found thus :—

$$\text{Major axis} = \sqrt{8^2 + \left(\frac{16}{3}\right)^2}$$

$$= \sqrt{64 + 28.44}$$

$$= \underline{9.61 \text{ ft.}}$$

Then the *area* of the ellipse of section is—

$$A_E = .785 \, Dd \qquad D = 9.61 \text{ ft.}$$

$$= .785 \times 9.61 \times 8 \qquad d = 8 \text{ ft.}$$

$$= \underline{60.3 \text{ sq. ft.}}$$

which agrees fairly well with the first result.

To measure the cutting and waste we must state the circumference or periphery of the ellipse, which we know from vol. i. $= \pi \times$ mean diameter (approximately).

$$\text{Mean diameter} = \frac{D+d}{2}$$
$$= \frac{9.61+8}{2}$$
$$= 8.805 \text{ ft.}$$
$$\therefore \text{ Circumference} = 3.142 \times 8.805$$
$$= \underline{27.66 \text{ ft. run.}}$$

Cutting and waste would be given as 28 ft. run fitting round circular chimney.

If the same principle is to be applied to areas of plane surfaces of various slopes standing over known plans, we have only to divide the plan into parts having the *same slope*, treat each part separately according to its inclination, and *sum the results*.

For example, find the area of the verandah roof of Fig. 138. Its ridge is 3 ft. 6 ins. above the eaves.

Note that the section against the 12 ft. wall, 4 ft. wide, would be made into a rectangle by the plan of the hipped end nearest you being fitted to it at the valley, for they both have the same slope.

Plan area of surface U + the hip $= 11' 6'' \times 4 = 46$ sq. ft.
„ „ „ S $= (10-4) \times 5' 6'' = 33$ sq. ft.

Slope of U is found from—

$$\text{tan of slope} = \frac{3\frac{1}{2}}{4} = .875$$
$$\therefore \text{ "angle" of slope} = 41° \text{ nearly.}$$

Slope of S is found from—

$$\text{tan of slope} = \frac{3\frac{1}{2}}{5\frac{1}{2}} = .6363$$
$$\therefore \text{ "angle" of slope} = 33°.$$

$$\text{Real area of "U + hip"} = \frac{46}{\cos 41°} = \frac{46}{.7547}$$
$$= 60.95 \text{ sq. ft.}$$

$$\text{Real area of S} = \frac{33}{\cos 33°} = \frac{33}{.8387}$$
$$= 39.34 \text{ sq. ft.}$$

and total area of roof $= 60.95 + 39.34 = \underline{100.29 \text{ sq. ft.}}$

EXAMPLES FOR PRACTICE

(*Inclined Areas*)

(1) Find the area of the spire roof of Fig. 136. The plan area, being a regular hexagon, is approximately 2.6×5^2. The angle of inclination of the roof is approximately 82°.

Answer, Roof area = 464 sq. ft.

(2) Fig. 140 is the plan of a roof whose pitch is 45°. Find the area of the roof in square yards. *Answer*, 94.26 sq. yds.

(3) Assume the inclination of the roof of Fig. 141 to be 60°. What area of tiling would be required to cover it, measured nett, as laid?

Answer, 13.68 squares or 152 sq. yds.

(4) Allow the roof of Fig. 142 to overhang 15 ins. at the eaves. If its covering be half pitch (tile pitch), what nett measure of tiling in squares of 100 ft. superficial is required?

Answer, 13.04 squares.

(5) The rise of the roof in Fig. 143 is 1 vertical in $1\frac{1}{2}$ horizontal. Determine the area of roof boarding, felt, and slating required in its construction, neglecting waste and allowances.

Answer, Pitch = 34°
Plan area = $952\frac{1}{2}$ sq. ft.
Roof area = 1149 ft. superficial
= $127\frac{2}{3}$ yds. superficial
= 11.49 squares.

(6) A skylight in a roof has to cover a plan rectangle $7' \times 4'$. The roof is half pitch. What is the area of the skylight?

Answer, 39.59 sq. ft.

(7) A circular lantern of 7 ft. diameter is to be erected to stand upon a roof of 40° inclination. What tile area will require to be subtracted from the gross measure of roof covering?

Answer, 50.21 sq. ft., or 5.57 sq. yds.

(8) The piece of ground represented by Fig. 189 is to be considered as giving the measured "surface dimensions" of the plot. It was found that the ground had a fall of 15 ft. across the breadth of the plot. What is the real area measured on a horizontal plane above the plot?

Answer, Slope = 4° approximately.
Horizontal area = 8782 sq. yds.

(9) Let Fig. 189*a* be the plan of a horizontal ground surface. Suppose it were now conceived to fall to an inclination of 11° across the longest median of the triangle, what is the increase of area?

Answer, Original area = 3199 sq. yds.
Inclined area = 3259.9 sq. yds.
Increase = 60.9 sq. yds.

CHAPTER XVI

ENLARGING AND DIMINISHING MOULDINGS—THE USE OF SIMILAR TRIANGLES

WE may now usefully turn our attention to another series of applications of similar triangles, showing their practical use in enlarging and diminishing architectural features, *e.g.*, mouldings, brackets, etc.

It is often desirable in working out the constructive details of buildings to enlarge a given shape in *one* or *both directions*, or to proportionately increase or decrease the dimensions of every part of a design. The reasons for this process may be any one of the following:—

(*a*) To obtain a "wider" or "narrower" moulding of the same character as a given one.

(*b*) To prepare a bracket of the necessary shape to allow a "mitre" of two mouldings to be prepared upon it.

(*c*) To determine the templet for application on a splayed surface to produce a required square section.

Our object in considering the geometrical process involved in the enlargement and reduction of mouldings, is not, however, to detail the particular circumstances causing the process to be necessary, but rather to illustrate and apply the principles, preparatory to their detailed use in more advanced constructional classes.

You are now fairly conversant with the properties of similar triangles, and these properties will be left (with brief references) for your recognition as we proceed with the applications.

Fig. 202 is an elevation of a rough bracket whose outline consists of straight lines only; it is required to obtain a bracket "wider" than the given one, but of the same depth at corresponding points in the outline—that is to say, *the "widths of similar parts" must have the "same ratio to each other" in each bracket*, which must be the ratio of width of the "given" bracket to the "required" one.

The elevation of the bracket is required first and the plan of its "face" which is the straight line AB. A and B lie on

the vertical projectors from the ends of the elevation; continue *one* of these projectors downwards, say beyond B. Take the required width of bracket in the compasses and with A as centre, cut the vertical through B at C. Join CA. Consider this line as the "plan" of the face of the required bracket.

Project a new elevation on this line as a ground line (observe auxiliary projection). Drop perpendiculars from all prominent points in the elevation on to the plan AC, and note that ABC becomes one of a series of similar triangles, all contained within it, each breadth along AB being reproduced to an enlarged dimension in constant ratio on AC, where the projector intersects this line. The ratio $\frac{AD}{AE} = \frac{AB}{AC}$, and so on, for each pair of dimensions.

Project from AEC at 90° to AC, measure the corresponding heights *a*, *b*, *c*, *d* from the primary elevation, and complete the figure which is *enlarged in width* only in correct ratio part for part.

This principle may be applied to mouldings and curved figures quite easily by selecting suitable points on the curves, finding these in the enlarged or diminished drawing, and connecting them by fair curves agreeing in nature with those from which they have been projected.

Fig. 203 illustrates this process. A moulding is given; we desire to increase its *thickness* while maintaining the same *width*. T_g is the "given" thickness and T_r the "required" thickness. Now observe that we might adopt the exact arrangement of Fig. 202, but we will rearrange the working so as to be more concise while applying the same principle. Draw the given moulding in full detail as per Fig. 203; enclose both mouldings by a rectangle, continue GH horizontally, make it *any* length, and join FH. (For convenience FH may be drawn with the set square, say at 45°.) Continue the horizontal through K, terminating at J on the perpendicular from H. Drop vertical projectors from prominent angles and intermediate points in the curves. Carry horizontal projectors from the same points to meet FH, drop these vertically to JF, and transfer horizontally again to *intersect the corresponding vertical projector from the same point*, thus determining the *similar* point on the enlarged drawing.

Complete the figure with extreme care, noting that circular arcs become elliptical in the enlarged figure.

Observe that FG is divided in the same ratio as FH, FH the same as FJ, and FJ the same as FK. Hence the heights along FG are divided, yet increased, in the same ratio as FK.

Observe, further, that such enlargement or reduction of a

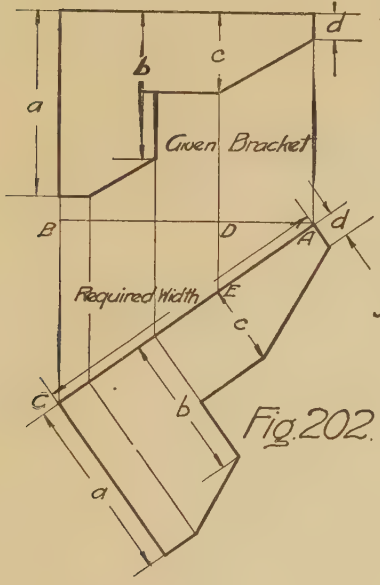


Fig. 202.

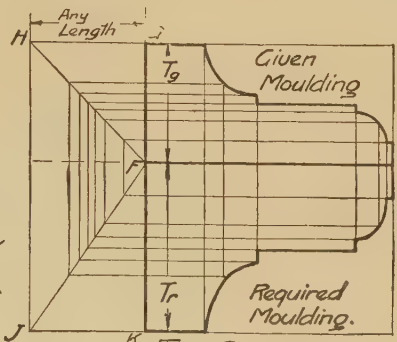


Fig. 203.

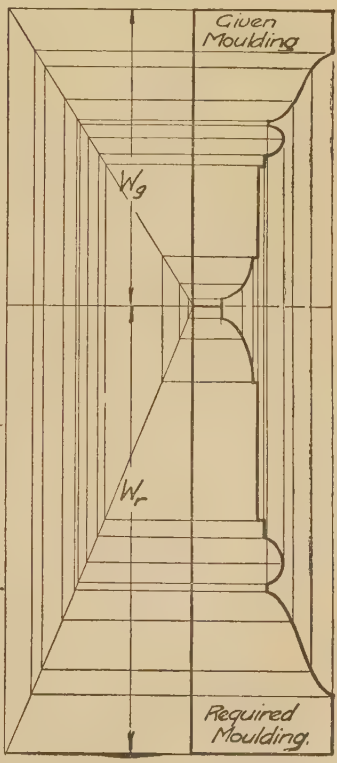


Fig. 204.

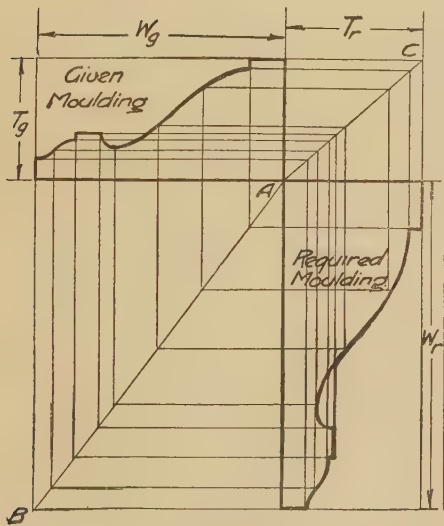


Fig. 205

figure may result in serious distortion of some parts of the moulding, so that, as an architectural feature, the result is bad. This is emphasised at Fig. 203, the mould determined being much too thick for the width, proving the design to be unsuitable for these dimensions. This is a matter for care and consideration, both in practical and professional work.

A similar application is displayed at Fig. 204, the width being enlarged while maintaining the same thickness. W_g is the "given" width, W_r the "required" width. Being a repetition of principles, identical in every way with Fig. 203, no further explanation is deemed necessary.

In Fig 205 we proceed a step further. Here it is proposed to produce a moulding similar in design to a given one, but to a *stated width and thickness.* Observe the arrangement carefully. The mouldings *given* and *required* are set so that their *widths* form consecutive sides of *one* rectangle and their *thicknesses* consecutive sides of a *second* rectangle ; the diagonals of these are AB and AC

By similar triangles formed about the diagonal of the larger rectangle AB the divisions of width are reproduced by projection —vertically and horizontally—and the thicknesses are similarly projected horizontally to the diagonal AC, and transferred by vertical projection. Vertical projectors from AC intersecting corresponding horizontal projectors from AB determine the similar points in the required figure—in this case "enlarged."

A little careful study should enable you to follow the application of the similar triangles and the mode of procedure.

Should it be desired to enlarge or to diminish a figure in "strict proportion"—both in width and thickness—then $\frac{W_g}{T_g}$ must equal $\frac{W_r}{T_r}$.

Suppose we have the given width = 3 ins., given thickness = $1\frac{1}{4}$ in., required width = 5 ins., find thickness of the required figure to be in strict proportion.

$$\text{Now } \frac{W_g}{T_g} = \frac{W_r}{T_r}$$

$$\therefore T_r = \frac{W_r \times T_g}{W_g}.$$

$$\text{Inserting values, } T_r = \frac{5 \times 1\frac{1}{4}}{3} = 2\tfrac{1}{12}''.$$

This value may be determined geometrically and incorporated so that the remaining process becomes identical with Fig. 205.

Fig. 206 illustrates it *without the mouldings.*

The W_g and T_g are set out in rectangular form as before ; PA is produced downwards, and W_r measured at AE. Produce

DA to the right and join DE. With the compasses measure T_g from AP at AF. Draw FG parallel to DE, then AG is the required thickness (T_r), for—

$$\frac{AD}{AP}=\frac{AD}{AF}=\frac{AE}{AG}$$

$$\text{or } \frac{W_g}{T_g}=\frac{W_r}{T_r}$$

Transfer AG with the compasses to the right at T_r. Complete the figure, using any well designed moulding "within the given

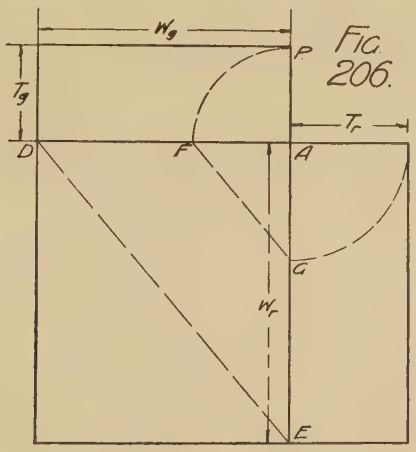


FIG. 206.

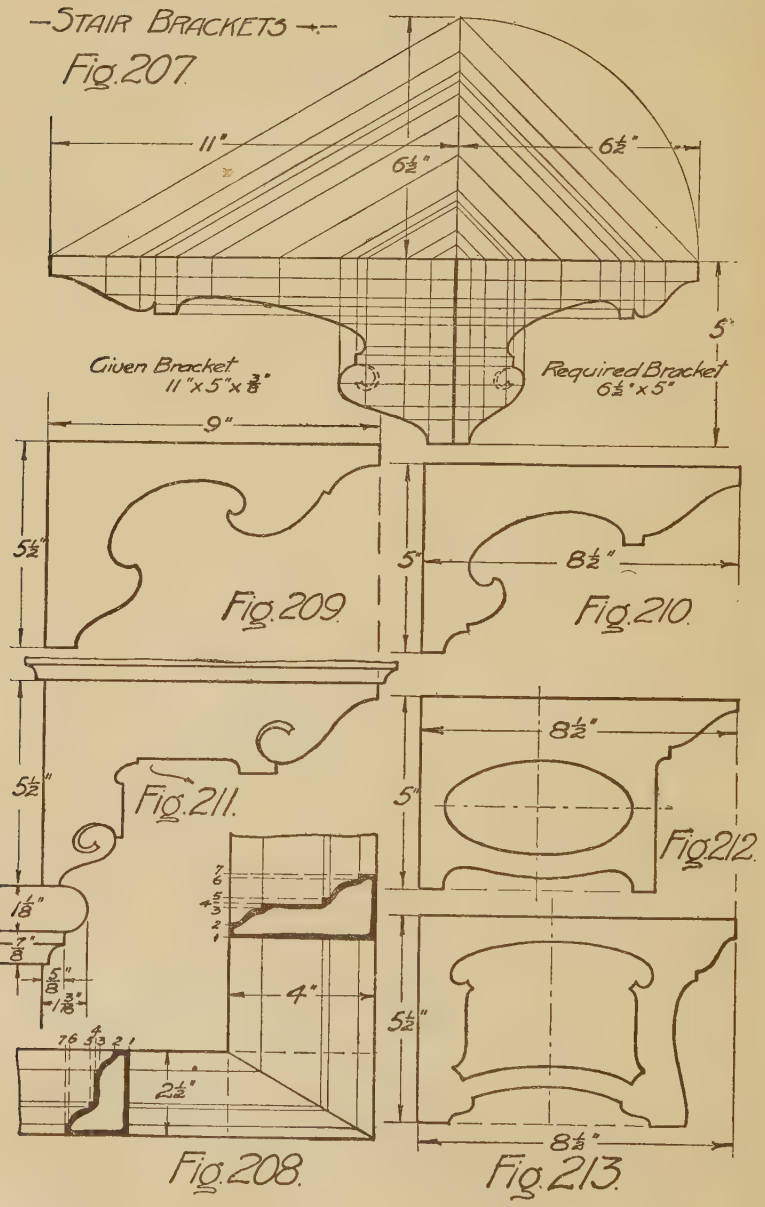
rectangle," and construct the proportionate figure within the rectangle you have just determined.

Fig. 207 shows a special application of the principles to the diminishing of a stair bracket, such as may be required for fixing at the reduced end of a winding step in a cut and bracketed geometrical stair.

After the previous explanations the figure should be self explanatory. It is probably the best arrangement for the easy solution to this common case. Study it and apply the method in the following set of examples where convenient.

Fig. 208 shows how two intersecting mouldings at 90° (such as a pair of architraves) may properly intersect though of different widths. One is 4 ins. wide and the other $2\frac{1}{2}$ ins. only. Let the wider one be given (*see* its section on the moulding).

Place the narrow moulding in elevation (at 90° across the length), draw the mitre line, add the elevations of the members of the given moulding to cut the mitre. Carry these horizontally, thus obtaining an elevation of the "edges of the members in the positions necessary to allow them to mitre." Project a


STAIR BRACKETS
Fig. 207
11"
6½"
6½"
5"
Given Bracket
11"×5"×⅜"
Required Bracket
6½"×5"
9"
5½"
Fig. 209
5"
8½"
Fig. 210
5½"
Fig. 211
8½"
5"
Fig. 212
1⅛"
⅞"
⅝"
⅜"
4"
2½"
Fig. 208
5½"
8½"
Fig. 213

cross section of the narrow moulding from these lines, making the thickness at each part to *correspond with the given section at a similar position.*

This method might also be employed to determine a moulding of the *same thickness* and a *different width,* exactly comparing in its *result* with that of Fig. 204.

EXAMPLES ON ENLARGING AND DIMINISHING MOULDINGS

(See *p.* 220 *for ideas of "forms of moulding" for special purposes, Figs.* 282 *to* 308).

(1) Take any architrave, say $5'' \times 2''$, and reduce it in correct proportion to a width of 4 ins., and a thickness of $1\frac{1}{2}$ ins.

(2) It is required to carry out a given design of cornice moulding right through the details of a structure. The original design was 5 ins. projection, and $4\frac{1}{2}$ ins. deep, and similar to Fig. 282. Reproduce it in correct ratio to suit a change of dimensions :—

(*a*) For a cornice $7'' \times 5\frac{1}{4}''$.
(*b*) „ $3\frac{1}{2}'' \times 3''$.

(3) A moulding for planting on a ceiling as an architrave round a manhole is shown at Fig. 215. Reproduce this and obtain a similar moulding suitable for an architrave to a doorway in the same room, to be $4\frac{1}{2}$ ins. wide, and strictly in the same ratio of width to thickness.

(4) Through a mistake made by a workman, an architrave moulding, similar to Fig. 214, which should have been placed round a doorway near the angle of a room, could not be got in. The architect instructed that a narrower, but properly proportioned, moulding should be inserted to mitre correctly with the top architrave. Its available width was found to be $2\frac{3}{4}$ ins. Design the reduced architrave.

(5) Choose any two of the designs of stair brackets, Figs. 209 to 213. Reduce *one* of these to $4\frac{1}{2}$ ins. wide, making your choice of design suited to the reduced width, and the *other* to 6 ins. wide.

(6) The whole of the stair brackets referred to in Example 5 have been gathered from practical cases—existing stairs—with slight variations in the proportion. Which of these brackets are most suitable for reduction, and which would not allow of *any* reduction without spoiling the design? What practical objections do you find to the design of Figs. 212 and 213? Enumerate them.

(7) Turn to p. 220, Figs. 282 to 308. Here you have a selection of mouldings for various purposes, and parts of common building details. Make selections from these, and apply the problems we have explained in the text.

Also try to enlarge and reduce these to required sizes by freehand drawing, maintaining both character and reasonable proportion.

NOTE. — A good designer may, and does, usually abandon all mechanical methods of enlarging and diminishing, except in very large detail, and the freehand moulding is generally the most effective and beautiful.

CHAPTER XVII

INTERSECTION OF MOULDINGS

It is now useful to consider the further relation of mouldings to each other where their position and form are such as to cause some difficulty to arise in providing for their correct intersection.

Mouldings may meet each other at a change of direction, while both lie with their principal surfaces in, or parallel to, one plane. Two panel moulds, two architraves, or any pair of similar mouldings laid flat on a surface and joined where they cut each other at the angle of a panel or doorway, are the simplest illustrations.

If they are both straight and alike in cross-section, the plane of intersection is a *straight mitre*, such as you can find in almost any example of moulded panelling or framing.

This "mitre" bisects the angle between the lines of direction of a moulding; therefore a mitre between two moulds meeting at 90° is at 45° to each direction and *at* 90° *to the plane of the moulding*. For any other angle the mitre line will be formed to divide the angle into two equal parts.

The outside edges, if allowed to meet in a drawing, cut each other such that the internal point of intersection is equidistant from each of the opposite edges, and is therefore a point on the bisector of the angle. (*See* chap. xx. on Loci.)

We must now find what happens when the mouldings do not run in straight lines.

Fig. 216 is an illustration of plane intersection, viz., where mouldings of the same general character are laid on a "plane surface." It represents the wooden soffit over the entrance to a shop doorway.

We are not concerned with the exact construction, though this is indicated, but with the form of panelling and the mode of arranging the mouldings round the panels and making them meet and *mitre* properly.

Examine these panels, five in number, which show square intersection at the left-hand stile, oblique intersection (as at B)

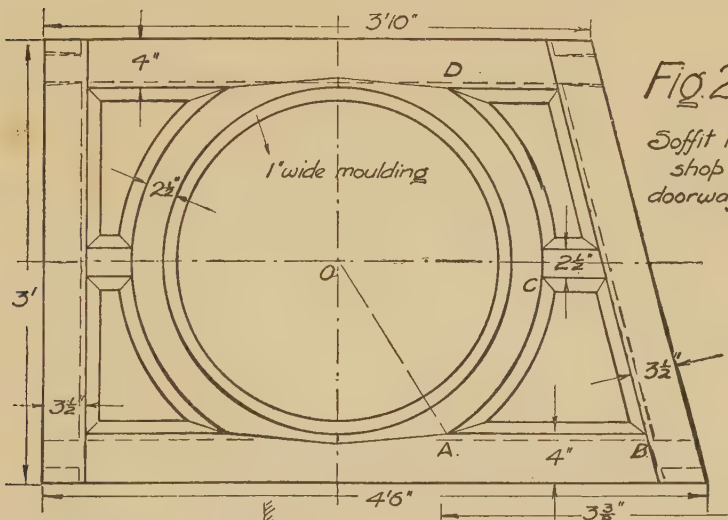


Fig. 216.

Soffit to shop doorway.

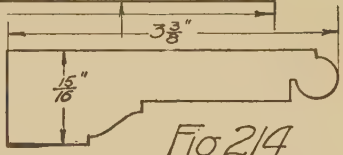


Fig. 214

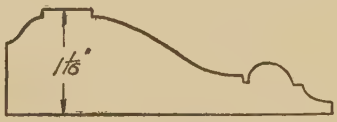
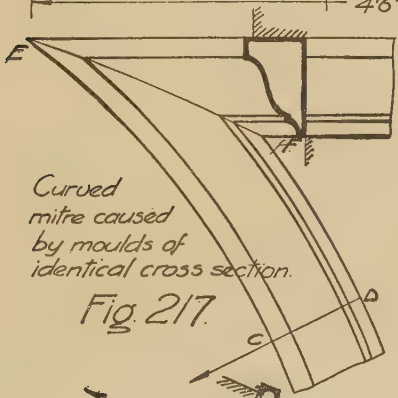


Fig. 215.



Curved mitre caused by moulds of identical cross section.

Fig. 217.

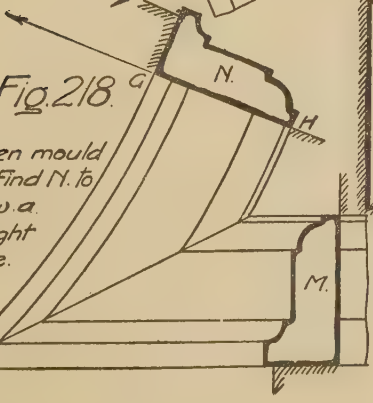


Fig. 218.

Given mould M. Find N. to allow a straight mitre.

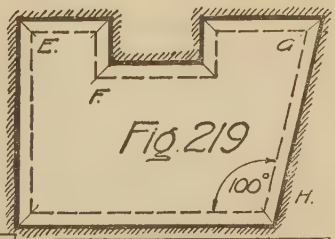


Fig. 219

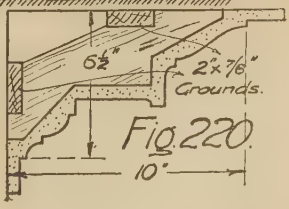


Fig. 220.

L

on the right-hand stile, and the intersections of curved and straight mouldings on the outside of the circular panel. *One* of these is almost equivalent to a right-angled intersection, but the other is fairly acute.

With two mouldings of the *same form* and *size* the angle at C could be mitred in a straight line with fairly satisfactory results, because it is "almost" square, but the mitre at A, if made straight between the intersections of the inner and outer edges of the mouldings, as shown in Fig. 216, would be a very deficient junction, because the members would not meet accurately.

To show this to be true, turn to Fig. 217, which is a detail at D. The straight length of moulding has its true section drawn upon it. The "same section" applies normally (on a radius) to the curved moulding, as at CD, and *the "spacing on this line" agrees with the "spacing of the members on the straight moulding."*

Set out a similar drawing with a different moulding. Carry all the members round the curve parallel to each other—from the same centre—and note their points of intercession between E and F. Lay a straight edge along EF, and you see at once that the meeting points of the members *do not coincide with it.* Hold up the page with Fig. 217 almost on a level with the eye, and look along FE. *The mitre is distinctly curved.* In your own drawing the mitre must be lined in freehand, or placed by bending a flexible straight edge to pass "fairly" through the points of intersection. A "fair curve" is *always possible* if correctly worked.

Where special mouldings are to be worked by hand, or much repetition to be done by the machine, and all intersections of straight and curved mouldings "are of the same kind and angle," it saves labour to make a "special moulding" to fit round the curve "that will mitre with the straight moulds in *straight mitre lines.*" The necessary alteration is usually so small as to be negligible in its elevation effect.

Fig. 218 shows how this might be done. It is a detail of a position similar to A in Fig. 216. The straight moulding is laid out on the panel, its section M and the plan lines drawn, and the straight mitre placed in position between the outside points of intersection; then the plan of the curved mould is projected by carrying parallel lines from their desired intersections at the mitre.

When a normal is now drawn across the panel as at GH the cuts of the members upon it do not agree with those upon the straight length. Construct the section at N, placing the members on the curves at the normal and measuring the thicknesses from corresponding points at M.

When intersecting mouldings are worked in plastic material their mitres are formed by letting each moulding continue its direction to meet the counterpart from the opposite direction, thus forming a natural intersection. This requires skill on the part of the workman, because the hand-worked mitre must lie in a plane at 90° to the surfaces on which the moulding is built.

So long as moderate dimensions of cornices and other plaster mouldings are adhered to, the whole may be formed in solid plaster, but if excessive in dimensions, such as large cornices and coved ceilings, the plaster "body" becomes impossible for several reasons (see your construction notes). It is then found necessary to fix brackets or frames against the main surfaces, fix laths on these forming the rough outline of the moulding, and form the latter with a comparatively thin structure of plaster.

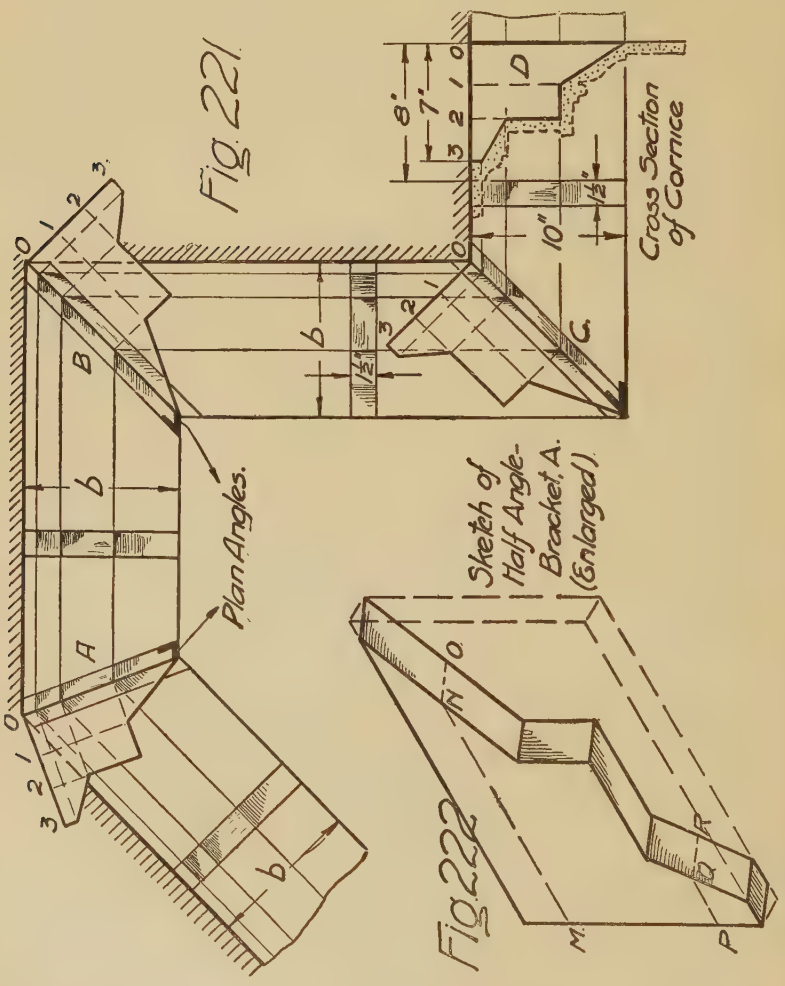
Fig. 220 is the section through one such cornice, which is probably the minimum size on which bracketing would be at all advisable. The reason for the application of brackets here is the "form of cornice adopted," which is of a classical outline in preference to the common coved cornice which is the economical dwelling-house pattern.

Fig. 221 illustrates the application of such a bracketed cornice to a room with a broken plan, the cornice altering its direction three times to follow the walls. The view is taken upwards towards the ceiling, and the position of the brackets ($1\frac{1}{2}$ ins. thick) are shown by shaded areas at the angles and intermediate points. Notice that "angle brackets" become a necessity for really sound work, because the end of the laths carried along the face of the brackets require to be firmly supported and fixed. But, as the plan of the mitre is "longer" than the plan of the bracket, while maintaining the same depth, the shape of the angle bracket differs considerably from the ordinary one placed at 90° to the walls. (Turn back to Fig. 202. The change of form of the "angle bracket" from the "ordinary" one is well illustrated by the difference between the "given bracket" and the "required" one.)

We employ the principle of Fig. 202 by noting the *plan* length, and combining it with the correct and constant depth at similar points on the outline.

In Fig. 221 the true cross section of the cornice is shown to the right, at D, dotted round the bracket whose shape follows roughly parallel to the cornice, *but in straight lines*. The plan of this bracket is close to the section on the left. Prominent edges are carried, in plan, round the whole of the walls, and these are seen to "mitre" at A, B, and C. A and B are "internal" angles, and C is "external."

Now if the brackets were plane figures instead of thick pieces of wood, the shape of the intersection on the real line of mitre would be all we should need to consider. The "centre line shape" is shown for each of the angles; these



are easy to follow from the projections and measured heights. Thus, each true shape is an auxiliary elevation of the mitre plane, or, if preferred, a rabatment into the ceiling plane.

Because the brackets must have thickness and also take ground-work laths in *two directions*, it is wise to construct them

of two pieces of wood, each bevelled in the correct direction to allow of the laths being bedded and fixed.

The plans of these "bevels" are shown at each angle by strong lines. Notice that the two pieces for the external angle must form an edge like a hip, but those for the internal angle form a V like a valley. Fig. 222 illustrates one half of the angle bracket at A. Two of these are fixed together by screws forming *one* bracket, which is much easier to make accurately than a "solid" bracket. Observe that the positions for testing the correct plan angle are in horizontal planes as MNO and PQR.

You will gain further acquaintance with these and similar constructions in more advanced classes in carpentry and joinery. Compare the process with that illustrated in a simple manner for a coved ceiling, vol. i., p. 109.

A moulding may be stopped against a plane surface either meeting it square or obliquely. If met obliquely it usually happens that the elevation of the outline of intersection is an important matter, because of its bearing on the architectural appearance of the adjacent parts.

A typical instance of this condition is the intersection of sill and cornice mouldings on bay windows and oriels, and of the corbelling which is necessary to support the "oriel," which differs from a "bay," in that it does not commence from the ground level.

Fig. 223 is the half elevation, half plan, and vertical section of an oriel window showing the corbelling and sill only. The plan is a part octagon, and the sides therefore slope at 45° to the main wall. The vertical cross section at the centre line shows the form of corbelling and window sill projecting from the wall at a distance *qq*.

From this section the complete plan and elevation of the mouldings must be made, and as these are horizontal, but follow the shape of the window in plan, their directions change at the angle A and form a common intersection, while the main wall end of the mouldings are the result of a plane intersection with the wall, the mouldings appearing to splay and butt against it.

Because the mouldings are of constant cross section all round the window we have the mitre line AB in plan bisecting the angle at A. Lay this out and proceed to complete the plan, showing all prominent lines on the moulding in section and one or two intermediate horizontal lines on the curves (*see* section). To do this choose the angles and points in section, drop to q^1q^1, transfer to *qq* on plan, and carry along parallel to the face on each side of the mitre AB, stopping

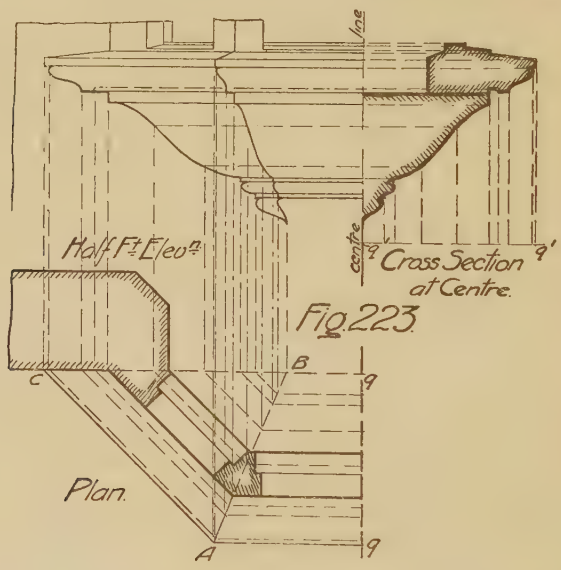
them at the wall plane CB. We are now in a position to determine the elevation of the mitred angle AB, and of the intersection with the wall, for we know the elevation position and plan position of the points on these intersections.

The process, therefore, becomes :—

(*a*) For mitre, project vertically from plan of same and horizontally from the section, and determine the intersection of corresponding projectors.

(*b*) For wall plane intersection, follow the same procedure projecting vertically from the plan intersections on line BC to the same series of projectors in elevation.

Now, by the aid of mental conception of the form of these mitres and intersections, draw in the outline of the curves,



knowing that they must be similar in character to the section but "elongated at the wall" and "depressed at the mitre." The illustrations should make this process clear, and also show what care in choice of section is necessary to give a pleasing elevation at the wall intersection. The moulding of the sill in *this* case gives a little too much prominence to the sill in the general elevation, though its sectional form appears good.

We are led by experience to sometimes determine or design the wall outline first, then, working backwards, obtain the true section from it. Neither method is "necessarily" successful,

and one must use the results of experience to determine the best mouldings.

For strength, the corbelling may be wisely treated as in the first sketch section of Fig. 226, where the upper part is the "reversa" curve.

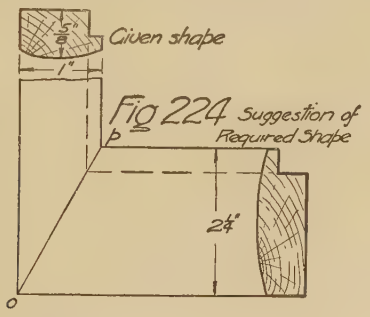
EXAMPLES ON INTERSECTION OF MOULDINGS

(1) Work out the form of each different angle bracket for the ceiling of the room shown in Fig. 219. Adopt the outline and detail of cornice illustrated in section at Fig. 220.

NOTE.—If this be satisfactorily completed you should be sufficiently conversant with the process, because four types of angle bracket occur in it.

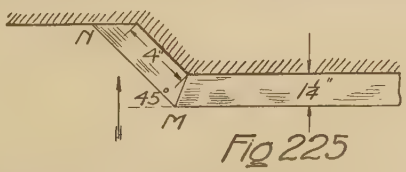
(2) A ceiling is constructed with a quadrant cove of 2 ft. radius, abutting at the head and foot on a suitable moulding, one on the ceiling and the other on the wall. Design the mouldings and cove, then determine the forms of the angle brackets required at the angles G and H in Fig. 219, and the bevels for construction in two parts. You may need your teacher's special aid here.

(3) Fig. 224 gives the section of a picture frame "stile" and the elevation of its mitre with a much wider "rail." Determine accurately the



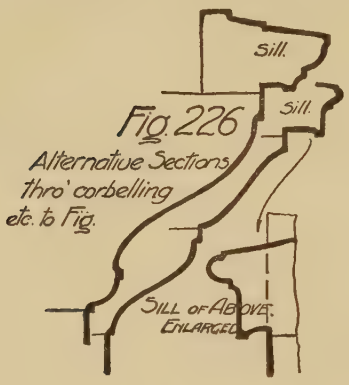
required cross section (suggested only in the diagram) to give a perfect mitre at OP.

(4) Fig. 225. A horizontal moulding over a doorway was returned round an angle as shown by this plan. Using the moulding of Fig. 230,



determine the elevation of the moulding, showing its mitre at M and its intersection with the wall at N.

(5) Choose one of the sections at Fig. 226 and adopt a similar arrangement of oriel window to Fig. 225. Determine the complete elevation of the sill and corbel. Add to the elevation the head of the window opening with a projecting moulding, as in Fig. 227. (The working



of the latter is identical with that of the corbelling.) Take great care in your proportion and draughtsmanship in reproducing these mouldings.

(6) Fig. 214 is a moulding in use as an architrave. The moulding is stopped at the base on a foot-block with a 45° bevelled top surface. Draw the elevation of the intersection.

(7) Show the elevation of the intersection of a planted moulding, $1\frac{1}{4}'' \times \frac{3}{4}''$, flush with the framing of a door, and stopping at its lower end on a chamfered lock rail bevelled at 45° to the panel face, and with $\frac{1}{4}$ in. of "square" on the edge of the rail.

CHAPTER XVIII

RAKING MOULDINGS

MOULDINGS are often required to meet each other in an intersection or mitre which, for the sake of appearance, must itself have some definite position. Such a case arises when a moulding placed horizontally on one face of a structure is required to meet another moulding running at an inclination across an adjacent face. The latter moulding is commonly termed a "raking mould."

We cannot, in this volume, deal with many cases of raking mouldings, but we shall endeavour to grasp the first principles on which such intersections are made in architectural practice.

Examine the sketch, Fig. 229. It represents a "horizontal" moulding on the left-hand face of a vertical wooden post, meeting a "raking" mould on the right-hand face. The two are so formed that they have *identical sections on the mitre plane*. Let us see what conditions will produce identical sections.

You must first satisfy yourself that *two mouldings of the same cross section, nailed with their flat backs against the sides of the post, "could not" be made to mitre properly*. The similar surfaces would not finish at the same lines, and overlapping of the pieces at the mitre would occur. Try this with pieces of moulding if you cannot visualise the conditions.

Now observe that if the horizontal member be canted over at the top edge until its *back* becomes at 90° to the slope of the raking mould (*see* Fig. 229*a*), it would then mitre perfectly, but the mitre plane would be "oblique" instead of vertical, and the whole mitre would have a very distorted appearance.

Examine Fig. 229 more closely. To obtain a perfectly satisfactory result we have to arrange that the following conditions are fulfilled :—

(*a*) If the mouldings are fixed to, or may be conceived to spring from "vertical surfaces," the mitre surface must lie in a vertical plane. (Call this the "mitre plane.")

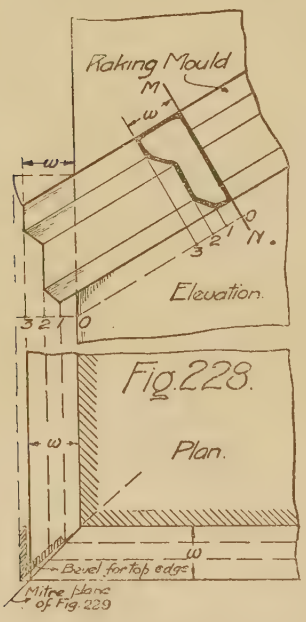
(*b*) If the plan thicknesses of the mouldings are to agree the "mitre plane" must bisect the plan angle.

(*c*) The true sections of the two mouldings must be such, that when cut by the vertical mitre plane, the sections made across the two mouldings are exactly the same.

We shall adhere in our examples to "90° plan angles." The conditions *a* and *b* above are necessary to preserve the same general position of the mouldings with regard to the planes on which they are fixed; *c* demands that the two mouldings shall be of different true sections, the difference being controlled by the position of the mitre and the angle of inclination of the raking piece.

We should now be able to understand the following statement: In arranging raking moulds to intersect with horizontal ones, one moulding must be *given* (or chosen) and the other made dependent upon it.

There is a choice as to which shall be the "primary mould," and this choice is dependent on further conditions such as, Which is the more important moulding? or, Which will be necessary in the greater quantity to complete the work in hand?



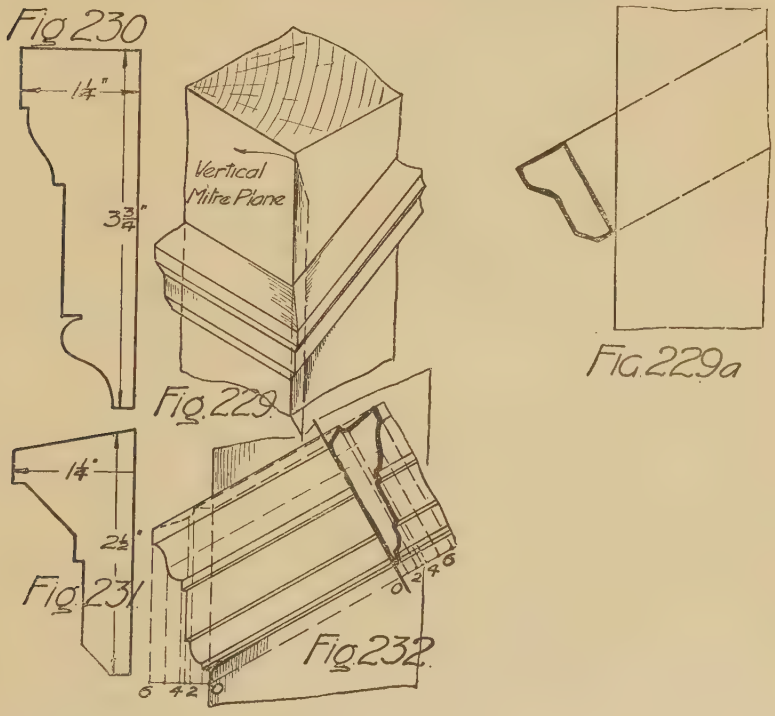
Generally, we might say that the best architectural result is obtained when the "raking" mould is made of the ordinary cross section (square edges, etc.), and the horizontal moulding designed from it.

In order to obtain conversance with the principles involved we have chosen in Fig. 228 a straight line cross section of a moulding, approaching in its shape some of the commoner outlines.

Consider the plan of Fig. 228. Because the plan angle must be "bisected" by the mitre plane, it follows that the projection *w* of the mould must be the same from each face. Hence draw the plan of the mitred mould as shown. On the elevation set out the true slope and depth of the raking mould, and on it construct the true section of this moulding, shown in strong lines. Think of this as a

section on MN rabatted about the back edge into the plane of the post.

Project the elevation lines representing the edges of the moulding, parallel to the rake and well beyond the angle. Now make the "horizontal projection" of "each part" in front of the return face to the left, to "agree with that of the corresponding part" in the section. Each line (or surface)



of the mould passes forward beyond the angle, until it attains the same horizontal distance from the post. This is necessary to cause each similar surface to intersect on the chosen mitre line in plan. Complete the elevation outline. In doing so, notice that all the sloping surfaces on the raking mould must have *similarly* inclined surfaces on the horizontal mould to intersect with—that is, *they must lie in the same inclined plane.*

The result shows that the true section of the horizontal moulding (which is the same as the elevation obtained, because we are viewing it endwise) is of similar general form to the given moulding, but with *bevelled edges* and chamfers of

different inclination. *Similar lines on the two mouldings are in all cases contained in the same inclined plane,* which is parallel to the top and bottom edges of the raking moulding.

Fig. 232 shows the result of choosing a horizontal mould of reasonable cross section and causing the raking mould to have a suitable section for mitring. The plan is omitted, being unnecessary to the working of this part of the problem. Examine this horizontal moulding. It is partly dotted at the top edge, because of the inclined piece covering it somewhat.

Prominent points and edges in the section have been projected downwards to a horizontal line, while inclined lines at the rake of the mould are carried across the face of the post. The *"depth" of the raking mould is obviously "less than" the depth of the horizontal one.*

To determine the cross section of the raker, observe :—

(*a*) That the square projection from the face must be the same at corresponding points.

(*b*) The inclined projection from the edges of the horizontal mould contain similar edges of the inclined mould.

Hence a cross section at 90° to the inclined moulding may be constructed by projection and measurement, the numbered projections assisting in the determination of the outline. After following the projection of the points in the "section mould" from the illustration, note the distorted form of the members, giving an unnatural appearance, especially at the top edge.

An important lesson may be learned from this result, viz.: the top edge of a horizontal moulding in this position must have a bevel of a greater inclination to the horizontal than the inclination of the raker, or at least "equal" inclination.

Fig. 233 shows the previous processes applied to two horizontal returns at the edges of a pilaster, when the section of the inclined moulding at *aa* is given and a more curved "moulding section" adopted.

Observe the difference between the "lower return" and the "upper." You see that "two special pieces" are necessary here. They are obtained on identical principles, therefore a little study and a worked example should make this case quite clear.

Return now to the plan of Fig. 228. In constructing the mitre for these two mouldings, seeing that the plan does not show the true shape of "either" top edge, we need to determine the angle of mitre that may be applied on the edge of the worked stuff. For this purpose a development is necessary. It should be clear that the "edge of the moulding" is a figure in an inclined plane, and we may, there-

fore, obtain its true shape by rabatting it into any horizontal plane. We have chosen to lift the horizontal mould about its top edge nearest the post. The elevation shows this "edge point" taken as centre and the sloping edge lifted into the horizontal; the "real width" is then transferred to the plan by projection, and the angle moved outwards at 90° to the hinge line. (*See* chap. x. on figures in inclined planes, if you need further assistance.)

The shaded angle is the bevel required to mark on the top edge of the return moulding, and its "complement" is the mitre for the other piece, for they both lie in the same plane, and the moulds are at 90° to each other.

You should now be able to obtain the "bevels" for any similar case.

A further example of the principle of raking moulds and requiring very careful drawing is given at Fig. 234, where a cornice and open pediment are detailed. The crowning mould of the pediment is a cyma recta, and this member is required to be carried up the pediment so that it mitres in the same vertical plane as the rest of the return end of the cornice at M.

The cyma at the horizontal return has been determined first to give a good "but not too extended" outline, and to harmonise with the rest of the cornice below it, which should be at once added (or drawn first).

Adopting the usual angle of 23° for the slope of the pediment, the rake of this is set out, the cyma return carried up the incline, with two intermediate points on the curve, and a cross section of the pediment constructed at 90°. Observe that the cross section at *ee* (like the return) is exactly repeated below *fd* at the pediment section. Then the cyma reversa is measured forward from this, point for point, as in Fig. 233.

To obtain the "top return" for the open pediment the whole moulding is treated as a return from some given line such as *gg* — according to the opening desired. Then, projections at 90° to *gg* must agree with those of the corresponding parts of the mould from *dd*, and measured at right angles to the respective lines.

See Fig. 233 for similar (though easier) application. The results of such raking mould problems as we have been discussing here should, if at all satisfactory, be pleasing in appearance. If distorted and ugly, they require rearrangement of the design, or possibly some other preferable mode of treatment which does not involve raking mouldings.

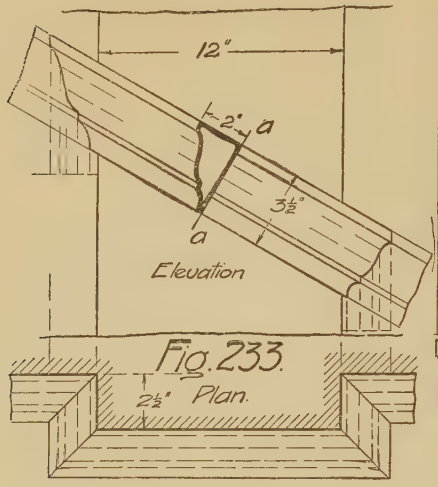


Fig. 233.

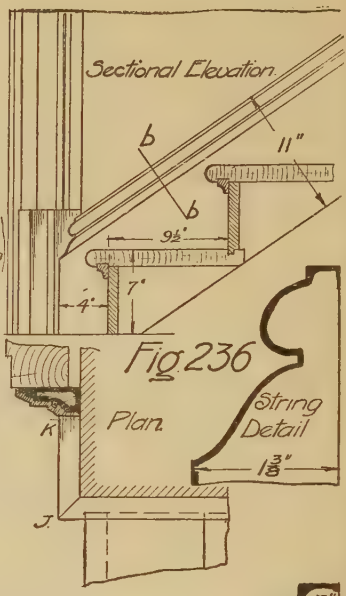


Fig. 236

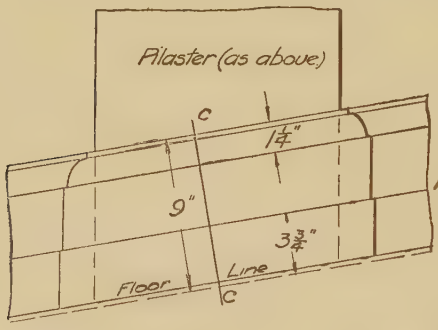
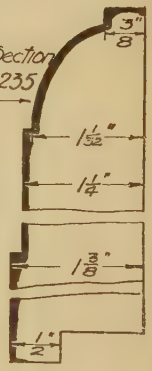


Fig. 235.



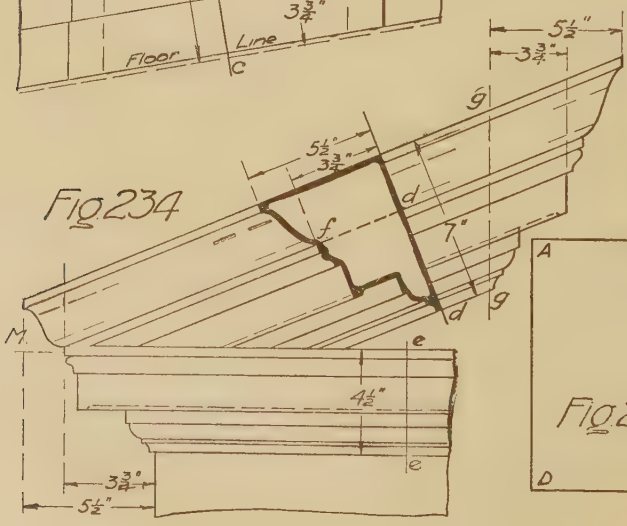


Fig. 234

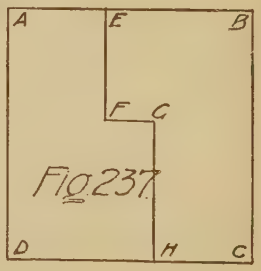


Fig. 237.

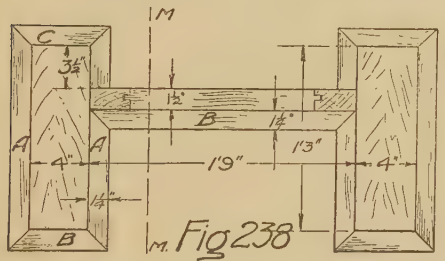
EXAMPLES ON RAKING MOULDINGS

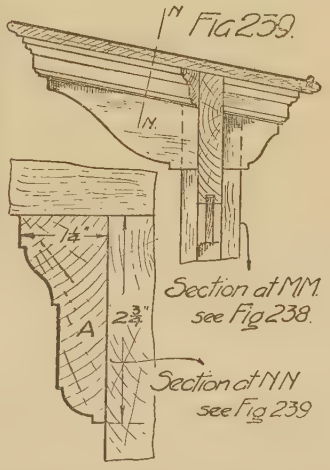
(1) Fig. 235 shows the elevation of a plinth whose cross section at CC and at each side of the pilaster is detailed to the right. The plinth is carried round, as shown in the plan immediately above, and the elevation outlines of the return pieces are indicated. Determine these "accurately" to a large scale—at least half full size.

(2) Fig. 236 is the plan and sectional elevation at the foot of a stair. The string is moulded, and the mould required to be returned horizontally to meet the plinth block of the architrave shown. (An easing cannot be used here because it would cause the return plinth to be too deep, rising above the plinth block.) Using the true detail of the inclined string mould shown, determine the exact form of the return mouldings. Scale, full size ; mould, 2 ins. deep.

(3) The plan of a room is given at Fig. 237. It is partitioned at EFGH. The ceiling of the room slopes at 30° from A upwards to B, and a cornice moulding of the shape shown in true section at Fig. 233 is carried along AB, FG, and DC. Find the correct moulding to apply at EF and GH on "both" sides of the partition so as to give true vertical mitres. Determine the bevels necessary at the "external angle" G.

(4) Plan of the *frame and supporting brackets* of a church reading desk is given at Fig. 238. A cross section at MM with the book board in





position is given in Fig. 239 to the right, and an enlarged section at NN in the latter figure is detailed below. The moulding marked A in the latter detail is used at AA in plan. Find the corresponding mouldings to apply at B and C to give true mitres.

NOTE.—Before leaving this section we should advise the student to work out all the book illustrations employing "different mouldings" under the "illustrated conditions" as well as the exercises set. Even then he will realise later that he has only got the main idea of the principles involved, and that the extended application and possible variations of the problems are innumerable.

CHAPTER XIX

ELEMENTARY COMPOUND CURVES

PREVIOUS work has shown us what a great part curves, such as the circle, ellipse, parabola, and tortuous curves, play in constructional work and architectural design. Extended practical experience will prove to you that the compounding of circular arcs plays an equally important part.

In arches and vaulting we shall often find that "compounding" is either a *necessity* or a *definite practical advantage*, but we must ensure that if such compounded curves are employed, they do not, through improper treatment, become defective and objectionable in appearance. As we shall find shortly, there are "limits" to the application.

Definition.—"An elementary compound curve is one continuous outline made up of *two or more* arcs of circles." Such curves are employed in setting out the outlines of large mouldings, approximately elliptical arches, Tudor arches, large sewers (egg-shaped), scrolls, and volutes.

From vol. i., chap. iii., you have probably learnt a good deal concerning the circle and its properties, and you know what is meant by a "tangent" and a "normal."

When two arcs of circles join end to end—if required to do so without a break, thus appearing continuous one to the other—they must have their point of connection on a "common normal," viz., a line which, if continued, contains the "centres of *both* circular arcs."

A tangent to either circle drawn at this point of connection is also "common to both arcs," lies in the common direction of each curve at the point, and the arcs are therefore said to join "tangentially."

Fig. 240 is an example of a compound curve consisting of four arcs of circles. It is "closed" as well as continuous. It represents the cross section of the inner curve of an "oval" (or egg-shaped) sewer.

Being a symmetrical outline we shall deal with "one-half"

of it. This sewer outline is most generally formed of parts of two circles tangentially connected (see dotted lines) forming the "invert" (lowest part of the curve), and the "crown" or cover (upper half circle).

Between these arcs a "connecting curve" is required, joining tangentially at the springing line of the crown, and at such points on the invert curve as may be possible. Choosing the cover to be semi-circular, the connecting curve will require to have its centre on the springing line produced, in order to lie on the common normal at the point of junction, D. Our problem is, therefore, to determine a point on the springing (produced) that will serve as a centre for an arc "tangentially connecting the primary circles." It must lie on a "common normal to both circles," and be "equidistant from each."

Method.—Deduct the radius of the smaller circle, *r*, from the larger one at DA. Join AC, and bisect it. Produce the perpendicular bisector to meet the springing line at B, which is thereby equidistant from A and C. But A is the same distance from D that C is from any point on the circumference of the smaller circle; therefore join BC, and produce to meet the smaller circle at E. Then BE = BD. Also E lies on the common normal containing C and B; hence B is the centre for *the only arc that will connect the circles as required.* Measure BC_1 on the opposite side of the centre line, draw the normals and repeat.

NOTE.—The two "primary" circles need not be in tangential contact. They may be quite apart, or again, may overlap each other; the construction still holds. We might note that the common practical form has the primary circles in the ratio of 2 to 1. Our principles are "general" and apply to all cases. For dimensions and further work *re* the above example, *see* the exercises at the end of this chapter.

Fig. 241 illustrates a large moulding composed of circular arcs. It produces a defective outline as arranged here, because the charge of curvature is too rapid, and without doubt a portion of a true ellipse would be more satisfactory. For small curves, freehand outlines judiciously proportioned are the ideal, because the personality of a good designer shows itself in the curves, but for large curves it is often necessary to draw mechanically to have any reasonable certainty of proportion.

The object of this figure is, therefore, to show the method of procedure. The curve must lie between A and B and

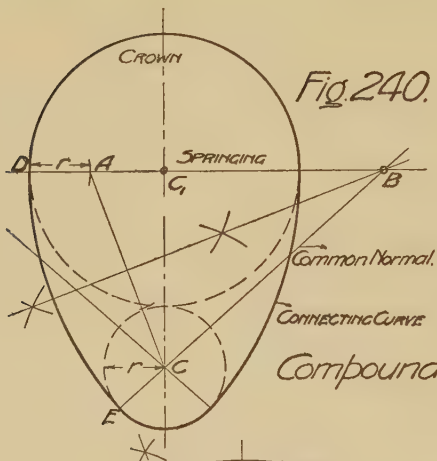


Fig. 240.

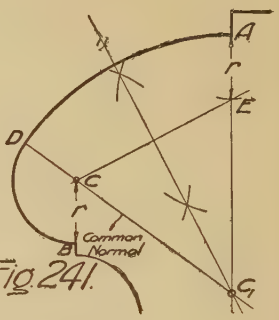


Fig. 241.

Compound Curves

Fig. 243. Approximate Ellipse

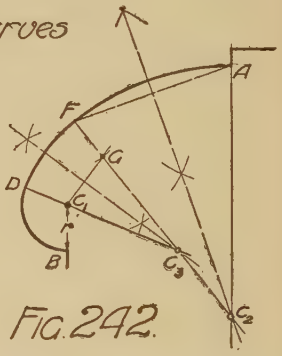


Fig. 242.

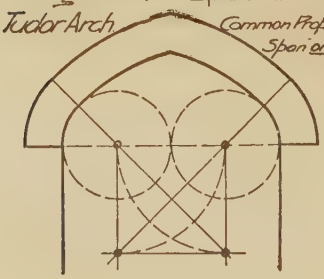


Fig. 244 Tudor Arch

Common Proportions. Span only given.

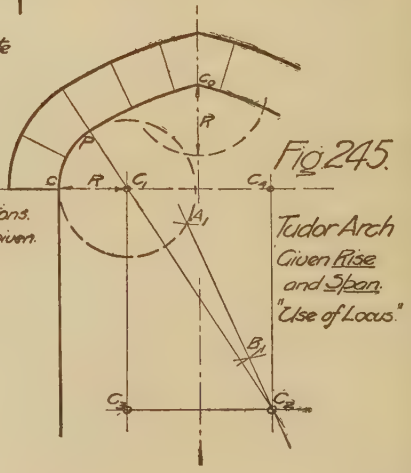


Fig. 245. Tudor Arch Given Rise and Span. "Use of Locus."

overhang at D. Choosing first the radius of the smaller curve which is thought to be reasonable set it off at BC, make AE parallel and equal. Join EC; bisect and produce to meet AE at C_1. Join C_1C and produce to D. Draw curve BD stopping at the common normal DC_1, then, with radius DC_1 and C_1 as centre, complete the curve to E.

Fig. 242 is a similar curve of different dimensions compounded of three arcs, so that the change of curvature is not so rapid.

This application is more difficult to carry out because you have a wider choice of radii. Suppose r in the figure to be settled (it may be made to agree with the curvature from B to D of a trial freehand outline). Strike the circle about C_1, going beyond D. Choose the point F at which you think the larger arc should stop. Join FA, bisect and produce to meet vertical from A at C_2. Join C_2F. Measure $FG = r\ (BC_1)$. Join GC_1, bisect and produce to cut FC_2 at C_3.

C_1, C_2, and C_3 are the centres required, and the arcs terminate on the common normals FC_2 and DC_3.

Draw arc FD first, and join the others on each side. Choice of position of point F is the vital matter, and calls for care and judgment.

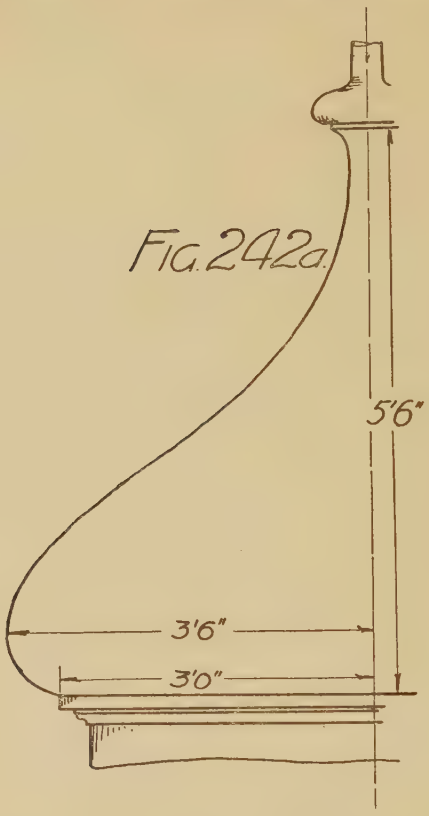
EXAMPLES ON THE PRINCIPLES OF FIGS. 240 TO 242.

(1) Draw the oval sewer of Fig. 240, when the larger base circle is 3 ft. diameter and the smaller one 1 ft. 6 ins. diameter. Other conditions the same as the figure. Outside the "inner curve" so obtained construct a parallel curve 9 ins. distant from it. Refer to your building construction book and utilise this drawing for a full detailed section of an egg-shaped sewer, showing concrete backing, terra cotta invert, brick body and crown. Get your teacher's advice if necessary.

(2) In Fig. 242 let the horizontal distance between AC_2 and B be $3\frac{1}{2}$ ins., and the vertical distance between A and B $= 4\frac{1}{2}$ ins. Assume the full projection of the curve from A to be 5 ins. Construct the curve compounded (*a*) of two arcs, (*b*) of three arcs, in a similar manner to Figs. 241 and 242. Note the defects, if any, and determine how they may be effaced or at least improved.

(3) Let the sewer section of Fig. 240 have the base circles (which are dotted) 3 ft. and 1 ft. 3 ins. diameters respectively, but placed 9 ins. apart on the vertical axis. Draw the inner curve of sewer section.

(4) Set out a roof turret curve, such as that indicated at Fig. 242*a*, using compound circular arcs. Adopt your own freehand outline, based on



the given dimensions, as a guide to the final curve. Show your construction clearly for the "centres" and "common normals."

There are several ogee curves in previous examples which are well worth your attention here as examples of the possible use of compound arcs as Figs. 157, 168, and 170. Employ these as foundations for further examples.

Arches

We must now enter on a subject which is more or less controversial, viz., the setting out of elliptical arches. No ellipse may be set out with compasses to be true in form

because the curvature of an ellipse varies from point to point in the curve, hence it is held by some that no attempt to utilise compound arcs is allowable; the resulting curve is never beautiful, and has no advantages to recommend it in practice.

Approximations by compound arcs are, nevertheless, necessary for economy, have many advantages due to capacity for repetition of the units, and may be of reasonably good outline as demonstrated in Fig. 243.

Should the arch be of small rise so that the ratio of the minor axis to the major is less than $\frac{2}{3}$, the simple method of Fig. 243 would produce an outline of less satisfactory shape, but even then we may, by employing five centres, obtain an excellent approximation to the true curve which, in large arches, may save a large amount of labour and trouble, because of the number of stones which may be cut to one templet. We must not, therefore, discard the approximate ellipse as a useless thing, but rather recognise its limitations when making our choice between the two in practice.

The "true" ellipse is, of course, the most beautiful, and where labour is of no consequence it should be adopted.

There are several "methods" of determining the approximate outline, but in most cases a "method" is only suited within certain limits, unless it takes the form of basing the desired curve on "sufficient points lying in the *actual* ellipse."

Our illustration in Fig. 243 (*see* p. 178) shows how this may be done in a simple way. Utilising what the author of vol. i. described as the "umbrella" method (more generally termed radial method) of construction (*see* p. 123, vol. i.) draw the major axis BB_1, the minor axis AA_1, and construct the enclosing rectangle BDD_1B_1 over the half ellipse desired.

Divide BO and BD into two equal parts, and repeat on each side of the axis. Join 1_1A, 2_1A, also A_11 and A_12, and produce to intersect 1_1A at E and 2_1A at E_1.

Then $BEAE_1B_1$ are all points on the true ellipse, and our desired approximation shall pass through these points. Bisect EA and produce to cut the axis at C_2. Join C_2E intersecting the springing at C_1. Repeat for the opposite side, obtaining C_3.

Observe that EC_2 is the common normal for the arcs of circles joining at E and E_1C_2 for the arcs joining at E_1. C_2 is equidistant from EA and E_1, and C_1 from E and B.

Place the centre arc in position first and fit the two smaller ones accurately to it.

NOTE.—It is useless attempting this construction unless accurate work be done in every detail.

For the parallel curve set out the depth of the arch at the crown above A and carry this round from the same centres, joining the arcs on the same common normals.

Joints in arch stones should always be arranged so that one joint falls on the normal at the change of curvature—points E and E_1—and the segments above and below these points so divided as to provide approximate uniformity in dimensions of "voussoirs," allowing a key block at the centre.

Keen students should go one step further and draw a larger arch by finding *two intermediate points* between B and A instead of one as in this case. They must remember that the point of junction of each arc lies on the common normal to the two curves joining thereat. *See* example 2 in the next set of exercises.

One more practical point occurs to us regarding this type of arch, which is, in setting out doorways and windows it is usually wise to choose the "stonework opening" as the *main curve*, and make the outer arch curve and the woodwork within the arch "parallel to the main curve."

Fig. 244 (p. 178) illustrates the arch known as the "Tudor arch," a depressed pointed form, each half being compounded of two arcs of circles. This figure gives the common method of striking it, producing invariably the same ratio of rise to span (about .43). The construction and procedure should be obvious without explanation if the diagram is studied.

We do need, however, some method of determining the curve when a definite rise is in view for a given opening.

Fig. 245 explains the first method of doing this. If you think over the problem you will recognise that we have a wide choice, for the springing arc may be varied largely in radius, causing a different crowning arc to be necessary for every change.

The method adopted here is based on the following :—

(*a*) Decide upon the radius of the springing arc R, which is wisely made not much over half the rise.

(*b*) Settle the vertical lines on which the centres for the crowning arcs shall be. In this case we have placed the centres C_2 and C_3 on the same verticals as C_1 and C_2, and on *opposite sides*.

Consider the inner curve—often called the "sight" curve. Draw the springing circle, radius $\frac{1}{2}$ to $\frac{5}{8}$ the rise. Let C_0 be the crown point ; about it describe a part of a circle, radius R.

Now observe that "if an arc of a circle having a given centre would pass through C_1 and touch the dotted crown circle tangentially," a "parallel circle, having a radius increased

by R, *would touch the exterior of the springing circle and pass through the crown point* C_0."

Find a point, therefore, equidistant from C_1 and the dotted circle about the crown, and also lying on the vertical passing through C_4. To do this the path of a point moving so that it is equidistant from C_1 and the circle, is determined. A_1 and B_1 are points in this path, and are found as follows:—

Take a radius = 2R and with C_0 as centre cut the springing circle at A_1. With any extended radius and C_1 as centre strike an arc, and with *this radius* + R a second arc to cut it at B_1. (B_1 may be *inside* or *outside* the line from C_4.) Join A_1B_1, and produce (if necessary) to cut the vertical at C_2, which is the required centre.

NOTE.—This path $A_1B_1C_2$ is called the "locus" of the moving point, keeping equidistant from C_1 and the dotted circle of the crown. *See* chapter on "Loci."

Join C_2C_1, and produce upwards to obtain the common normal and point of junction of the curves, P. The outer curve is obviously parallel to the inner, and drawn from the same centre.

Fig. 246 (overleaf) is a rather well-known method of setting out the Tudor arch to a given span and rise. Its "form" may be varied with the choice of height AD in relation to the required rise CE. The given proportions are arbitrarily chosen.

The method is as follows:—

Set out span and rise AB and CE. Erect ED = $\frac{5}{8}$ of EC. Join DE and set EF at 90° to it, produced downwards. Make AG and EF both equal to AD, and draw arc AK with G as centre. Join GF and bisect, producing the bisector to meet EF prolonged at J. J is the centre for the crowning arc. Join JG and produce to K, which is the point of junction of the two arcs.

Transfer the positions of the centres to give symmetrical arrangement, and complete the curve. The crowning arc is very flat, and "distinctive" because of this.

Observe that the principles of tangential connection have again been satisfactorily applied.

EXAMPLES ON ELLIPTICAL APPROXIMATIONS AND TUDOR ARCHES

(1) Set out an approximate form of elliptical brick arch 9 ins. deep, span 5 ft. 6 ins., rise 2 ft. Show the joints of the brickwork, remembering that one joint must lie on the common normal at the junction of the arcs, and that the maximum thickness of a brick is 3 ins., as showing

on the face of this arch. Add a parallel curve within the brick opening $1\frac{1}{2}$ ins. distant from it, to represent the sight edge of a moulded door frame.

(2) A stone arch has a span of 25 ft., and a rise of 7 ft. 6 ins. Using "five" centres (two intermediate points on the true ellipse between the springing and the crown) describe the elevation of the arch, 1 ft. 6 ins. deep, with square stepped extrados jointing into 10 in. ashlar courses.

(3) Set out the elevation of a Tudor arch 5 ft. span and 8 ins. deep, assuming that the rise is not defined. Show centres, joints in the stonework, and a key or crown block. Scale not less than 1 in. = 1 ft.

(4) Using the method of a locus determine the elevation of a Tudor arch, given that the span is 6 ft., rise 2 ft. 3 ins., radius of "springing arc" $\frac{2}{3}$ of the rise. Show your construction clearly, and represent the joints of the stonework in an arch 10 ins. deep. Do you find anything objectionable in the proportions of the curve? If so, what has caused them?

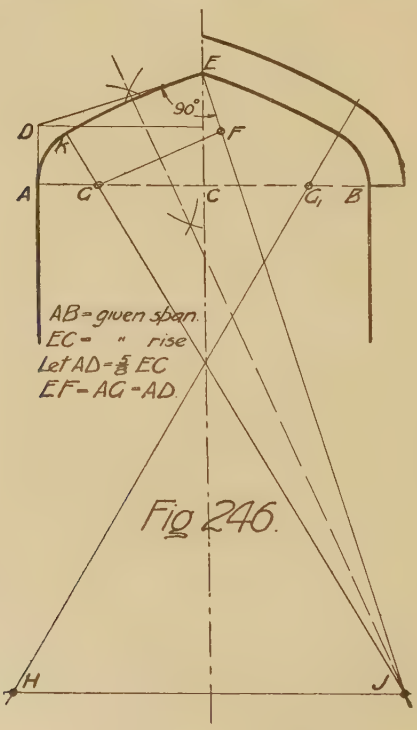


Fig 246.

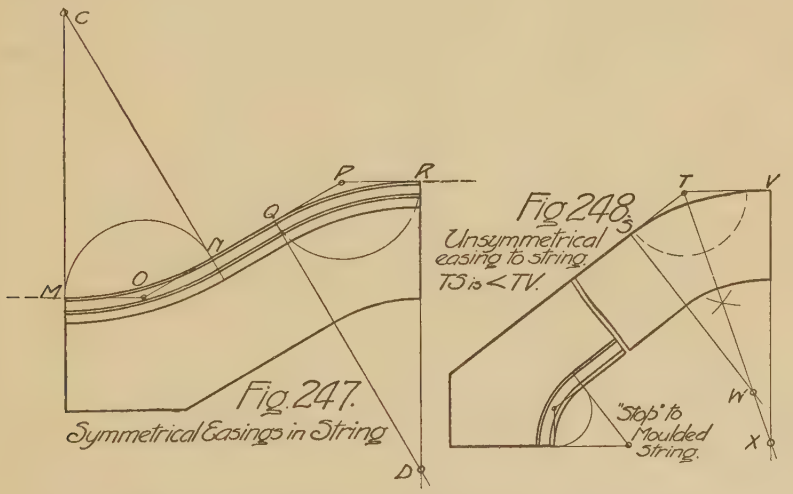
(5) Apply the method of Fig. 246 to the same arch, but reduce the radius of the "springing arc" to $\frac{5}{8}$ of the rise. Again discuss your results. What ratio of rise to span is apparently suitable for this form of arch?

Answer, $\frac{1}{3}$ or a little over.

Easings and Similar Curves

Curves are often required to connect with straight lines for the purpose of avoiding abrupt changes of direction, as, for example, in stair strings and handrails. In most of these cases the designer has to decide the amount and nature of the easing for himself.

The common method of deciding this is to settle the height of the string at a landing or floor, where the "easing from the raking string to the level" is required, and to make the curve of a suitable length for the purpose. Easings vary from 6 ins. to 10 ins. in length in ordinary cases, but there is no reason why longer easings should not be employed if the architectural value of the curve is not diminished. The method of arranging the easing is shown in diagrammatic form at Figs. 247 and 248.



The first figure illustrates the raking string with a moulding on its edge, changed in direction from the rake to the horizontal by easings at top and bottom ends, so that the same section might be continued as a plinth at the floor or landing level.

Consider the base, and suppose we are given the height of the plinth to join up at M. Produce the level line through M until it cuts the straight top edge of the string produced at O. On each side of O equal lengths are measured at OM and ON, and as the curve must join tangentially at M and

N the "normals" at these points must contain the required centre. Hence draw NC and MC at 90° to NO and MO respectively, to intersect at C. C is equidistant from M and N because the easing is symmetrical, and is the required centre. The moulding may now be described parallel to the top edge of the easing.

The easing to the upper end (for a landing) is exactly the reverse of the lower, hence should be perfectly clear.

Fig. 248 illustrates at the top end of a string an unsymmetrical easing, viz., one in which the lengths of curve on each side of the point of intersection of the two directions at T are unequal.

Let ST and TV be the lengths of the easing on the two sides of T, S and V being the points of junction of the arcs with the straight. (This condition may arise owing to a landing plinth being required to stand much higher above the landing than the string above the stair.)

The easing curve may be obtained, (*a*) by bending a flexible uniform lath through the points regulated until a fair tangential curve is obtained, and (*b*) by a compound curve of two circular arcs. The latter is the better method, and is particularly useful in drawing.

> *Method.*—Bisect STV at TX. Drop a perpendicular at S from the string and from the landing level at V. These do not meet on the bisector TX, but cut at separate points W and X. W is the centre for the lower arc, and X the centre for the upper one, and TX is the common normal through the point of junction.

Observe that STW and VTX are similar triangles, and the inscribed curves bear the same ratio to each other in position and radius as the lengths of the sides of the containing triangles. The bottom curve of the string is in this case made parallel to the top edge.

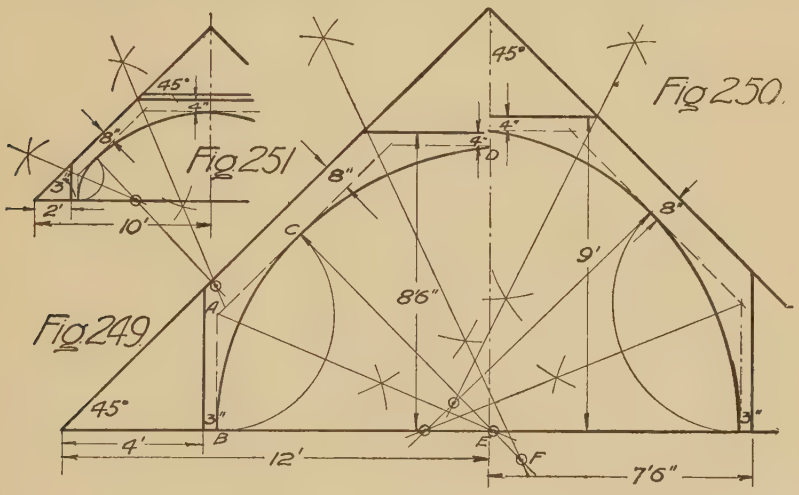
A further application of the easing, quite common in cut string stairs, is shown at the foot where a curved stop is inserted to bring the mould down square to the floor, which is an improvement in the finish.

In later years you will meet with many applications of this principle in both handrails and strings.

Before changing our subject we shall note one more case of the use of a compound curve, employed in providing curved braces in open wood roofs of the modern type employed in school and chapel roofs.

Fig. 249 illustrates the basic outline of half a roof where the rafter, tie-beam, collar, and a vertical strut are shown in single lines, and may be conceived as the "inner edges" of the members concerned in our problem. Our purpose is to provide a curved brace of a suitable pointed form within the centre area, which when complete gives an arched appearance and having a specified thickness (or depth) of material at certain points.

Noting these sizes (which are marked on the drawings), place dotted lines parallel to the edges of the members at the given distances, then proceed to construct a curve springing squarely from the tie, to touch these lines tangentially,



except at the highest point, which we desire to be the crowr of a pointed arch.

Bisect the angle A and produce to cut the tie-beam level at E. Make AC = AB; CE is, therefore, at 90° to the line AC. With E as centre describe arc BC. Bisect CD and produce to meet the normal CE continued at F. FC is the common normal required at the junction of the arcs. From F as centre continue the curve BC from C to D.

You will see that, according to the shape and size of the figure containing the "brace" its curve will vary much in character. Fig. 250 shows this clearly. The vertical strut is here made longer and the collar shorter, and the working is identical with Fig. 249.

A further illustration is provided at Fig. 251, where another change of dimensions has been made in the opposite direction to the change in Fig. 250.

The vertical strut is very short and the collar longer in proportion. Note and compare the results. Case I. gives a curve approaching a quadrant; case II. (Fig. 250), a sharply pointed arch; case III. (Fig. 251) a depressed arch of the Tudor type.

EXAMPLES OF EASINGS AND ROOF BRACES

(1) Set out the above three cases of "roof bracing" to scale, adhering to the dimensions supplied in Figs. 249, 250, and 251, and working without the aid of the book after reproducing the outlines. Now construct any other similar case, quite regardless of the dimensions, and place a similar bracket (or brackets).

(2) A collar beam roof, span 28 ft., has its main rafters at $\frac{1}{2}$ pitch. Its collar is 6 ft. above the springing of the rafters at the wall face, and wall posts, standing out $4\frac{1}{2}$ ins., are continued $4\frac{1}{2}$ ft. below the intersection of the walls and main rafters to a supporting corbel. Set out one half of the elevation of the roof members concerned, and determine a main curve for the braces, such that they are $4\frac{1}{2}$ ins. thick at the corbel, $7\frac{1}{2}$ ins. at their junction to the principal rafter, and 4 ins. at the centre of the tie beam.

(3) Determine the easing of a string to a plinth at the foot of a stair when the $\left\{\begin{matrix}\text{tread}\\\text{going}\end{matrix}\right\}$ is 10 ins. and the rise 7 ins., and the string rises $3\frac{1}{2}$ ins. square off the nosing line. Use the moulding shown in detail at Fig. 236 or Fig. 303, and make the plinth 9 ins. deep. Let the easing be 8 ins. along each tangent, and the arrangement symmetrical.

(4) Rearrange the example of Question 3 when the easing is unsymmetrical and measures 6 ins. along the "string tangent," and 9 ins. along the "plinth tangent."

(5) Fig. 252 illustrates a footstone at a masonry gable. Set out a "similar" elevation to scale, making the block containing the stone 2 ft. 9 ins.

long, 1 ft. 10 ins. deep, with the moulding at the eaves projecting 6 ins., and the coping stop $1\frac{1}{2}$ ins. The pitch of the roof to be 30°. Arrange the easing—

(*a*) Circular and symmetrical.

(*b*) Compound circular and non-uniform in length of tangent.

The amount of curvature is left to your judgment, and will depend partly on the moulding you choose.

(6) Set out the crown stone at the V of the same gable when the opposite side slopes at 60° as in a north-light roof. The sharp angle is to be suitably rounded over in the form of an unsymmetrical easing.

CHAPTER XX

LOCI

SOME of the work to follow requires a knowledge of the meaning and use of a "locus"—plural "loci."

It will therefore be convenient, and we think wise, to describe it briefly here.

We shall not have an "extended" use for this type of problem, hence our treatment will be briefly descriptive and illustrate the common and useful cases by examples.

Definition.—A "locus" is the *path of a point* moving according to some mathematical law, or traced under definite guidance of an unvarying nature.

The simplest examples we meet are :—

(*a*) The path of a point moving so that at any instant it is equidistant from "two fixed points."
Result—the perpendicular bisector of a line.

(*b*) The path of a point moving so that at any instant, it is equidistant from "two fixed lines."
Result—the bisector of an angle.

(*c*) The path of a point travelling at a constant distance from a fixed point in a plane.
Result—the circle.

(*d*) The path of a point moved so that the "sum of its distances from two fixed points" is *constant.*
Result—an ellipse ;

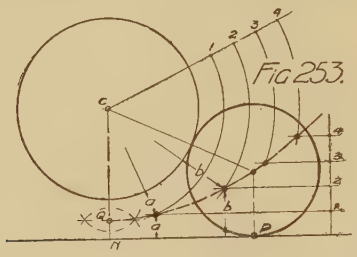
and there are many other cases in which we have only a passing interest.

Think over the above—if you have not previously studied them—sketching their conditions and making sure you understand them. The method of describing the ellipse defined by paragraph (*d*) above is the "string trammel" method you have previously employed (*see* vol. i., chap. v., p. 123).

The problems for which we require the application of a locus are chiefly those in which centres for circles in contact with others (or with circles and straight lines) are to be located,

when no direct geometrical method appears to be feasible, or you have forgot the orthodox method. Such problems occur in curved braces to collar and hammer beam roofs and in tracery work. We will take a few easy cases before illustrating their applications.

(1) Given a circle (centre C) and a straight line, Fig. 253.



Required, a circle to touch the straight line at P and also the given circle tangentially.

It should be easy to grasp from the figure—

(*a*) That the centre of the required circle must lie on the normal through P.

(*b*) That the centre must also be normally equidistant from the straight line and the given circle.

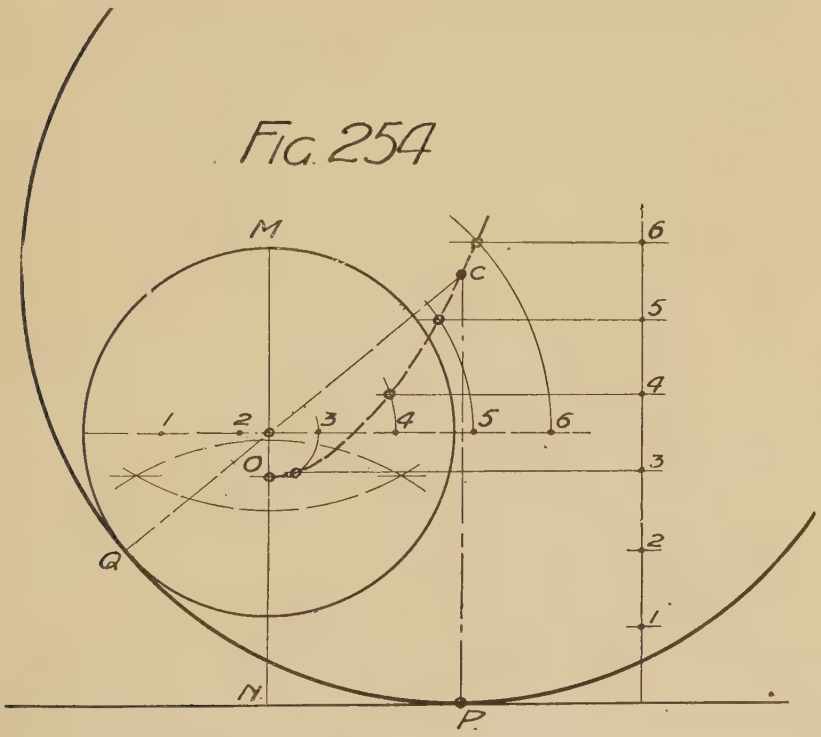
Method.—Find a few points which fulfil the condition (*b*) equidistant from line and circle. First such point is obviously Q midway between them lying on the common normal to line and circle. Second point lies on a line distant *a* from the given "line," and on a circle distant *a* from the given "circle." Third point is distant *b* from each, and this process may be prolonged until at least one point to the right of P has been obtained. Now draw a fair curve through these points, and if you test it you will find that it contains all such points as those we have been locating.

It is the locus of a point moving so that it is always equidistant from the given line and circle. Drop a normal to the line from P to intersect the locus. This is the required centre, and the radius is evident.

Please note that a more "direct" method of working the same type of problem is given at Figs. 263 and 263*a*, but the value of the above working lies in the simple principle involved which is often remembered when special methods are forgot.

Returning to Fig. 253, observe that another solution to this problem is possible, for a circle may be drawn to touch the line at P while touching and "enclosing" the given circle.

Its solution by a locus is shown in Fig. 254. This locus is conceived as the path of a point constantly equidistant from



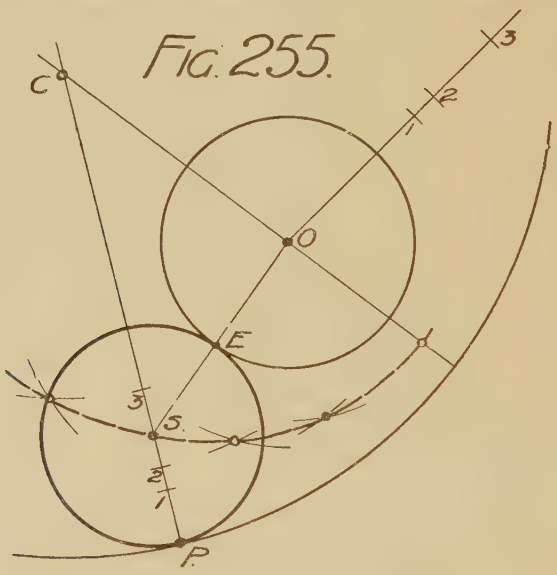
the line, and "the circumference of the given circle on the opposite side." Thus the first point of the locus is on the common normal MN and midway between these points at O. Follow the setting out of the "path" carefully; it locates isolated points equidistant from line and curve, the increasing distances being marked 1, 2, 3, etc. It is well that you should follow this example through without a detailed explanation until you arrive at the centre C. Observe that CQ is the common normal at the point of junction of the two circles.

NOTE.—A direct application for this problem is given at Fig. 263*a*.

A very similar problem occurs where the required curve is to join tangentially and make contact with "circles" instead

of the "circle and line." The two circles for contact may be so variously placed and conditioned that a large range of applications may be met with. They all conform to one principle in using a locus, therefore one illustration will probably serve.

Let Fig. 255 represent the conditions. Two circles (or arcs) are given, centres C and O. We require a circle to touch these two, the larger circle at a given point P. It is evident that the required centre S must lie on the radius CP, therefore construct the locus of a point moving equidistantly

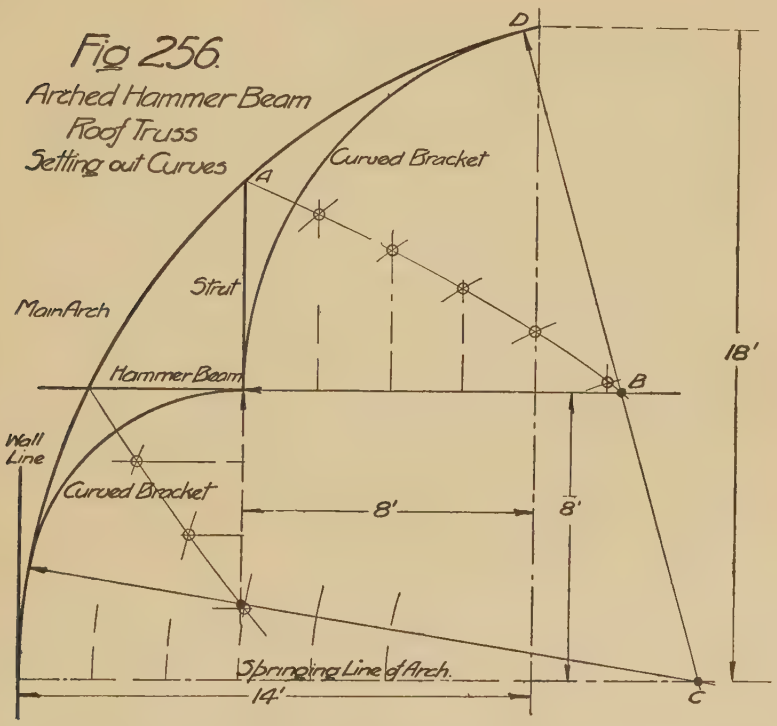


between the circles and away from them. This locus is shown, with the method of setting it out. Its intersection with CP gives the required centre S.

The arched hammer-beam truss affords a good example of the use of loci. Fig. 256 illustrates the form of roof in single lines; it needs careful examination. The truss consists of a large arch (marked "main arch") with a vertical strut and a horizontal "hammer-beam" carried inwards from the arch towards the centre. The main arch finishes against a wall post (marked "wall line"). In addition to our diagram outlines, a sloping rafter passes from the wall end of the hammer-beam to the ridge of the roof, clearing point A, and a level collar connects the "pair" of rafters and meets the arch at D. Do not trouble about the exact form or the

method of construction. These will come later. For the present our problem is to place curved stiffening brackets, "one" between the springing of the arch and the hammer-beam, and a "second" from the end of the hammer-beam to the main arch near the crown. Deal with the upper curved bracket first.

To spring properly from the hammer-beam it must have its centre on the horizontal line at the hammer-beam level.



Notice that we do not know either the radius of the bracket or the point of contact near the crown. Hence a locus is necessary. Determine this by commencing at A and locating a series of points equidistant from "arch" and "strut." (You might commence further from the angle than the diagram shows, when conversant with the resulting form of curve, but in the first case it is well to draw the complete locus from A to the hammer-beam level. You then see the type of curve which the locus traces; it is easy to mistake this for a straight line if two or three points only are determined.)

The process we have followed is: Draw a series of straight lines parallel to, and at known distances from the strut, and a series of curves parallel to and at the same distances respectively from the arch. Corresponding intersections then determine "points on the locus." Trace the fair curve until it intersects the level line containing the centre, at B. Through B draw a radius of the main arch CBD, meeting the arch at D—BD is the "bracket radius," and D the point of tangential contact.

Treat the lower bracket in a similar way, choosing that the centre for this bracket shall be on the line of the vertical strut continued downwards.

We could multiply this type of problem very largely, but probably superfluously, for one principle is involved in all problems involving "loci," and we shall now leave the working of further examples to "practically conditioned problems." (*See* chap. xxii. on "Tracery.")

Important.—Before leaving "loci" it will, perhaps, be well to point out one important matter, viz., that problems having only one unknown quantity to be determined, *e.g.* "a radius," may generally be solved by a "direct," therefore speedier method than the locus, and for such cases we advocate direct problems if the student has had the opportunity to study them; but there are many problems involving two or more unknowns, *e.g.*, "radius and points of contact," and in many such cases the locus is the only available method of solution, unless one falls back to the so-called method of 'trial and error." The latter may be justifiable in small scale work, and is, indeed, largely employed by designers of tracery work.

EXAMPLES ON "CIRCLES IN CONTACT"

(*Use of Loci*)

(1) Draw a straight line and a circle of 3-in. diameter, $1\frac{1}{4}$ ins. clear of the line.

(*a*) On one side of the given circle (lying between the given line and circle) place a second circle in contact with the two, touching the line at a point P, 2 ins. from the common normal to the given line and circle.

(*b*) On the opposite side, with the same conditions, place a circle to "enclose" the given one and tangential to the straight line at Q, 3 ins. from the common normal to the line and given circle.

(2) Place two circles of 3-in. and 2-in. diameter respectively on your drawing paper, $1\frac{1}{2}$ ins. apart.

(*a*) Place a circle in contact with both of them, one at a given point P, which you may place anywhere on the smaller circle. Your result must not enclose either of the given circles.

(*b*) Construct a similar circle, in contact at some chosen point on the larger circle, and "enclosing" the smaller one.

(*c*) Find a condition to enclose both circles.

NOTE.—Care must be exercised in the choice of the point P. In (*a*) P must not lie outside the common external tangents to the two circles, otherwise enclosure of one or both circles is necessary.

(3) Refer back to Fig. 245, where a locus was employed to determine a centre. Assume a span of 8 ft. and a rise of 3 ft. Let the radius of the springing arc be 1 ft. 9 ins., and the centre for the crowning arc be on a vertical line 2 ft. from the edge of the opening. Determine the inner curve, and make the external one 10 ins. distant. Introduce suitable stone joints.

(4) Complete the drawing of Fig. 256 completely to the dimensions given, showing your "loci" construction clearly. These loci may be curtailed as much as possible now that you are acquainted with their form.

(5) Take the following conditions for the arched hammer beam roof:—span, 30 ft.; rise of main arch, 19 ft. Intersections of hammer-beam and vertical strut with the main arch to divide it into "three equal arcs." Determine the curves for the brackets under similar conditions to the last problem. Does any difficulty occur with the "top bracket" regarding its junction at the crown? If so, how may it be overcome without spoiling the outline? Discuss it with your teacher.

HINT.—"Stilting" of an arch is employable here.

CHAPTER XXI

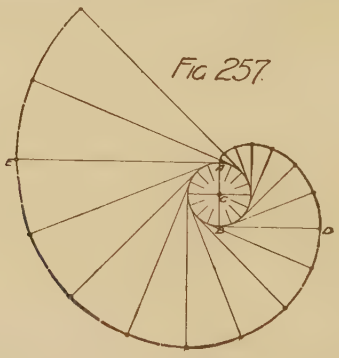
SCROLLS AND VOLUTES

A SPECIAL application of compound curves must now claim our attention for a while, viz., the description or "generation" of scrolls and volutes by circular approximations.

Before detailing these approximations it may be wise to obtain some clear idea of the meaning of the term "volute."

The two words "scroll" and "volute" are used more or less to convey the same meaning in architectural work, but the author prefers to differentiate between them, as will be seen later. Accepting the "volute" as the basic idea, it may be described as the result of a process of "unrolling."

Fig. 257 will convey the idea. Here is a circle, centre C, and a curve expanding about it, from A. It is conceived to



have been generated by unwinding a thread from the circle (or cylinder), commencing at A and keeping the thread "taut" during the process. The "locus" of the end of the thread at A has produced the expanding curve, which is known as the "involute of the circle" AB.

You now see why we studied the idea of a "locus" before entering on this chapter. This involute is the result of a "point moving under definite guidance of a constant character."

Fig. 257*a* represents another form of "volute" — though the application of the name to this case is doubtful. It is a curve traced by a point "moving so that it is constantly increasing its distance from a fixed circle during its revolution round the same, the increase being a constant amount for the same *angle traverse*." The curve is again a "locus." Its construction should be fairly obvious.

Now our object in representing these volutes as "loci" is not for you to draw them, but to point out that neither one

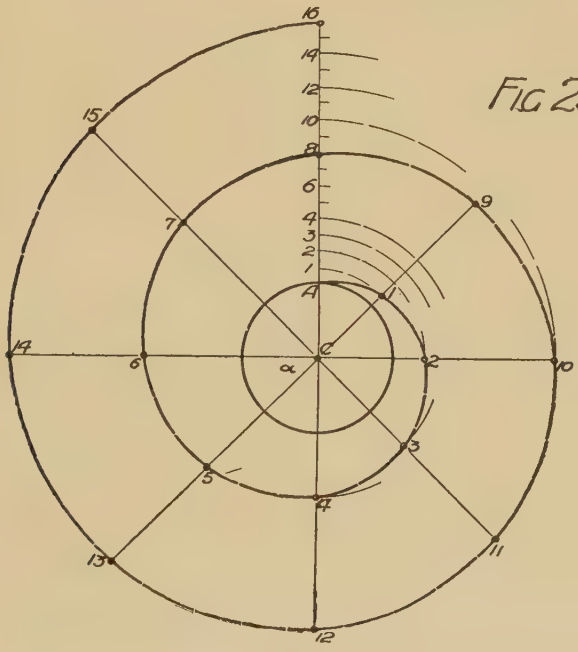


Fig 257a.

in its present form is suitable for architectural employment. The reasons are—

(*a*) That both volutes start more or less abruptly from the central circle.

(*b*) That 257 would, further, be difficult to arrange of any desired size, though easy to describe.

(*c*) Fig. 257*a* would be difficult to set out, though its size could be easily arranged.

(*d*) Neither of them suit our ideas of proportion, and would be difficult to so modify as to please.

In setting out scrolls and volutes we, therefore, resort to

either purely freehand work or to "geometrical approximations based on compound curves." We are concerned with the latter processes.

To save any further trouble in the use of terms we shall adopt the following:—

The name "volute" will be confined to an expanding member, where the main band or surface of the member expands as it recedes from the centre. The term "scroll" will be applied when the expanding member preserves the same width of surface.

You will observe that the first condition applies to the classical ornament peculiar to the Ionic Capital, and known as the Ionic Volute. The second condition is common to handrails, forming a stop at the foot of a stair, and known as "handrail scrolls."

If the curve be a "single line" expansion we cannot very well distinguish between them.

Approximate Volutes

The approximations we shall adopt have the same "mode" of expansion as a true volute, in that the curve is gradually increasing its distance from the centre of the figure, and such differences as occur in the practical methods of setting out these curves are generally differences in the "rate of expansion."

Generally, we shall adopt a combination of circular quadrants of increasing (or diminishing) radius, for one advantage of the approximate curve is the adoption of some overall width, or diameter, of the volute which is a distinct "practical" advantage.

We shall explain first the method of describing a single line volute, springing tangentially from a straight line and having a given "projection."

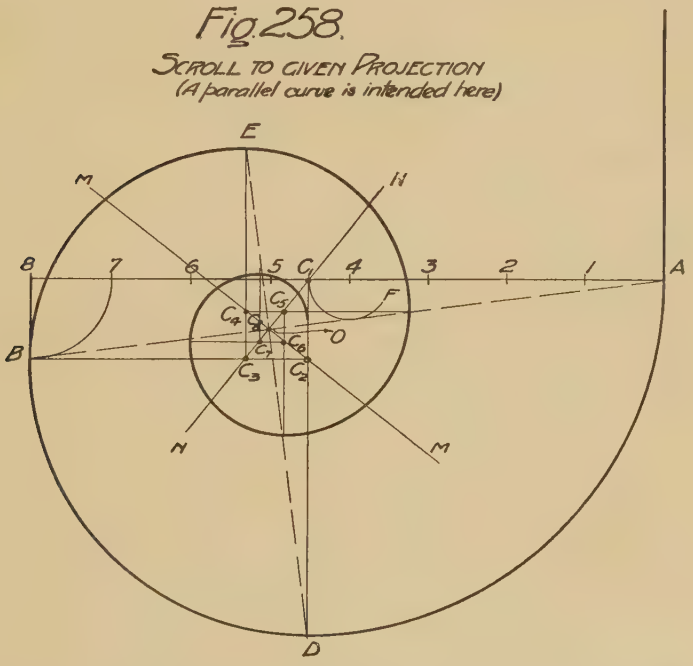
Fig. 258 illustrates such a volute. It consists of quadrants successively connected and of constantly diminishing radii, the "rate" of reduction being a pre-determined ratio involved in the "method."

Let AB be the "diameter" or "projection" of the volute measured at 90° to the straight line. Divide the diameter into 8 equal parts. Set off 8B at 90° to A8 ($\frac{1}{8}$ of the diameter). Join BA. BA is a "principal diameter" of the volute. We require a "second principal diameter" at right angles to the first which is determined thus: take the middle point of 8A (viz. 4) as centre, and draw arc FC_1 tangential to the principal diameter AB. Drop a perpendicular from A8 at C_1D, and with C_1 as centre, and C_1A as radius, draw the

quadrant AD. Place DE at 90° to AB. Then DE is the "second principal diameter," and O (the point of intersection of AB and DE) is the geometrical centre of the volute. It is the point towards which the centres of all arcs recede as the quadrants are added, and to which the volute convolves.

NOTE.—One "turn" of the curve during its expansion is termed a "convolution."

Join C_1O and produce, obtaining the line NN. Place MM through O at 90° to NN. Then MM and NN will contain



all the centres of the arcs required, for they bisect the angles between the principal diameters.

The determination of the centres is now accomplished by successive projection at 90° from C_1 to C_2 (on MM) from C_2 to C_3 (on NN), C_3 to C_4 (on MM), and so on.

The termination of each arc is obtained by making the line through each pair of centres the common normal for the adjoining arcs and making them meet thereon.

For example, the second quadrant DB is joined to the first one (AD) at D, because C_1 and C_2 both lie on a common normal to the two arcs at D, and the junction occurs tangentially.

The whole arrangement of principal diameters and centres appears as if rotated bodily through a given angle 8AB, which is really the case, and we could prove that the rate at which the volute approaches the centre is determined by this angle.

As a test for the student, he may, by experiment, verify that if this angle be altered by dividing A8 into *more* or *less* parts than 8, and the same working followed the volute approaches the centre at "quicker" or "slower" rate, according to the ratio of 8B to 8A in the diagram.

If 8B be less than $\frac{1}{8}$ the diameter, the approach is slower, and *vice versâ*.

This principle of constructing the curve is the best known and most generally useful method as a basis for handrail scrolls, the scroll being completed by a series of quadrants parallel to those of the single line volute. It is intended that Fig. 258 shall become a "scroll" by adding the parallel curve (*see* examples).

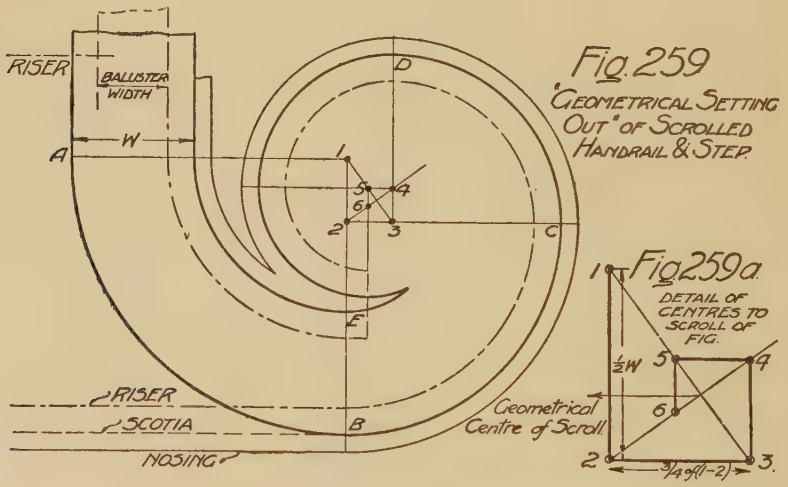


Fig. 259 is a special size of scroll intended for use at the foot of a stair with a 10 in. going (riser to riser). Its dimensions have been determined experimentally to suit this case.

The external outline of this scroll is a continuation of the inside (or stair) edge of the handrail, and is a volute of "similar form" to Fig. 258 with a little more than one convolution. To complete the solid scroll a curve is drawn parallel to the volute until the two meet. We have, therefore, to concern ourselves chiefly with the proportions and setting out of the external curve.

Assuming the "going" of the stair to be 10 ins., and the handrail width (W) to be $3\frac{1}{2}$ ins., then the scroll may be satisfactorily arranged by adopting the following conditions:—

(*a*) Let the outside edge of the scroll stand immediately over the "scotia line" of the step (probably $\frac{3}{4}$ in. in front of the riser line).

(*b*) Make centre 1 such, that the first radius—1.A or 1.B—is $2\frac{1}{4}$W.

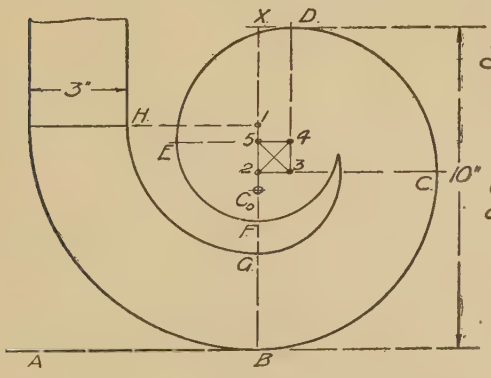
(*c*) By a series of right-angled projections and measurements make 1-2 = "$\frac{1}{2}$W," and 2-3 = $\frac{3}{4}$ of 1-2 (or "$\frac{3}{8}$W"). *See* Fig. 259*a* for a detail of these centres.

Then proceed to determine the rest of the centres by joining 3-1, placing 2-4 at 90° to it, and projecting 4, 5, and 6 by parallel lines terminating on the diagonals.

Observe that 2-4 and 1-3 are similar lines to MM and NN in Fig. 258, and contain the centres of all the arcs.

You are not concerned here with the construction of the "handrail scroll," or the "scroll step" below it, but you might note that the step, its scotia and nosing, are all made to follow the handrail scroll. The dotted line in plan is the step riser line, the "thick" line, the scotia and handrail, and the "fainter" line, the nosing of the step.

In the construction of the handrail scroll at the foot of a stair, the rail generally begins to rise for the "ascent" from the joint BE. On the right of this line the scroll is flat.



Now refer to Fig. 260, which shows an apparently simpler method of drawing a similar scroll. It is given here because it is a common process, but is not recommended as a general basis. Further, it is faulty in plan, because the approaching curves on the "inside" are almost parallel for some distance.

In a flat scroll this would be a visible defect, but in a "wreathed scroll" rising from GB towards H this fault is difficult to detect.

The method of describing is as follows: AB is the riser line of the first step. The "going" is 10 ins. Draw BX 10 ins. long, square to AB. Find its centre C_0. Make $(C_0\text{-}2) = \frac{1}{2}$ in., $(2\text{-}5) = 1$ in., and $(5\text{-}1) = \frac{1}{2}$ in.

First centre is	1	and radius		B-1.
Second	„	2	„	B-2.
Third	„	3	„	C-3.
Fourth	„	4	„	D-4, etc.

The inner curve is drawn parallel until intersection occurs. The resulting scroll has more "opening" within the scroll than the former one in Fig. 259. In a plain rail this may be an advantage, but with an elaborate moulded rail the author prefers the proportions of Fig. 259, because the mitre of the mouldings carries the "apparent" opening much beyond the the meeting point of the external lines of the scroll.

Carpentry and joinery teachers will begin here and detail the step construction, etc., in higher grade specialised work.

The Ionic Volute

We must next apply our volute approximations to setting out an ionic volute whose importance and character you will study in connection with architectural history and design at a later stage.

Our intention here is to explain one mode of setting out the volute and *persuade you to take sufficient care in attempting its completion* to the instructions.

The complete "volute" consists of a central "eye" or circle, about which a curved band is traced by rotation, gradually expanding in width until it reaches its maximum after three convolutions (*see* Fig. 261).

To grip the principle consider the single line of the volute forming the outer rim of the flat fillet, which is clearly followed in the drawing. (The volute is usually a moulded band, but we will think of it as flat for the present.)

AB is the diameter (depth) of the volute, which stands in a vertical plane. It is divided into 8 equal parts. The "setting-out eye" is $\frac{1}{8}$ of the diameter, and is placed so as to coincide with the fifth division from the top. The "setting-out eye" usually coincides with the "real eye," but in this case they have been separated for clearness, the former being "dotted," and the latter a full circle on which the receding curve terminates tangentially.

To obtain the centres proceed as follows:—
Within the eye (dotted circle) place a square with its diagonals

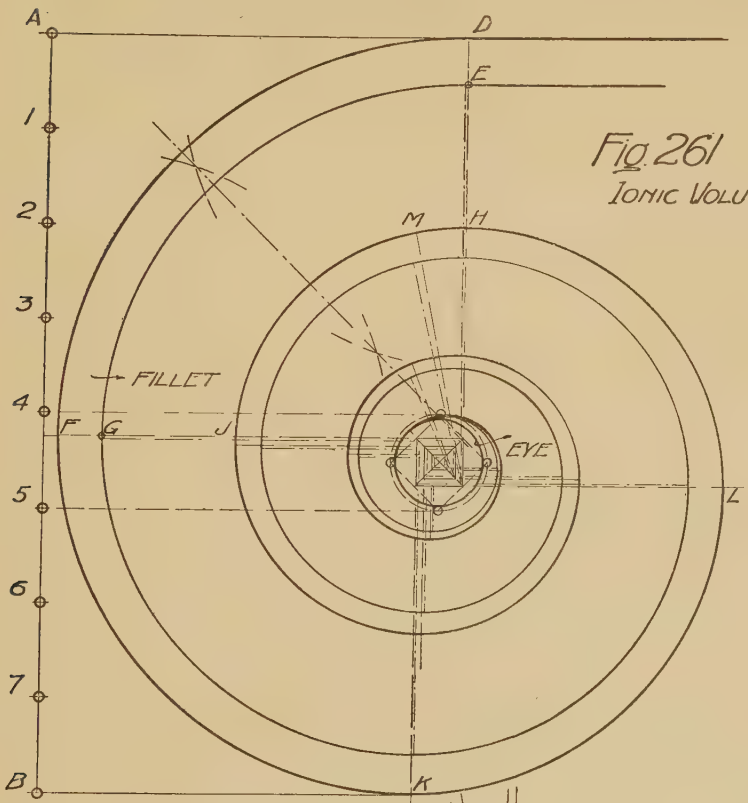


Fig. 261
Ionic Volute

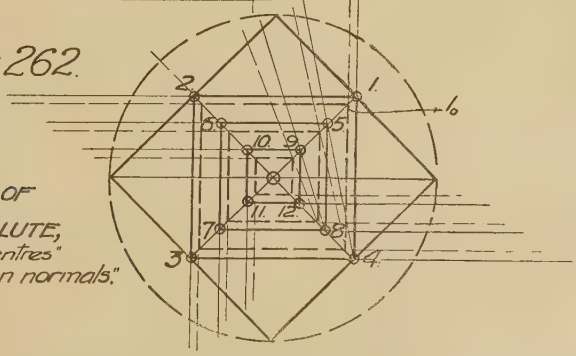


Fig. 262.

Detail of "Eye" of Volute, showing "centres" and "common normals."

vertical and horizontal. Its angles are clearly indicated by small circles in Fig. 261. Within this square place a "second

square," whose angles lie on the mid-points of the sides of the first square at 1, 2, 3, and 4 (*see* detail, Fig. 262). These are the centres for the "first convolution." Call the centre of the eye O. Draw the diagonals 2-4, 1-3. Divide half the diagonal into 3 equal parts, and project these divisions parallel to the sides of the square 1, 2, 3, 4, so as to form two inner squares equidistant from each other, whose angles are 5, 6, 7, 8, etc., up to 12. (Refer to both Figs. 261 and 262 for notation, please.)

The outer line of the volute may now be drawn thus: Take centre 1, radius 1-D, and draw arc DF terminating on 1-2 produced.

From centre 2, radius 2F, draw arc FK terminating on 2-3 produced. Repeat similarly until centre 4 is reached. Then observe that with 4 as centre and 4L as radius the arc LM is drawn to terminate on 4-5 produced. The reason for this extended curve is, that *it must terminate and the "next curve" begin* on a common normal containing the centres of the arcs joining at M.

If this geometrical necessity be not observed considerable difficulty will be experienced in continuing the volute into its second convolution.

Proceed as before, taking the centres in the numbered order until centre 8 is employed. Then join 8-9 and produce, making the arc with centre 8 *terminate* at this line, and the consecutive arc (centre 9) *commence* on it. The third convolution is similarly treated, and should finish neatly at the eye as shown.

To obtain the internal rim of the volute which shall accurately terminate in the same position at the eye, great care is necessary both in determining the centres and in setting the compasses at the junction of the arcs. A new set of centres is required having the same relation to the previous set as the breadth of the fillet has to the initial distance between the points DH of the first convolution.

The most accurate method of determining the point 1_0, Fig. 262, is to proceed as follows (Fig. 261): Let the fillet be $\frac{1}{4}$ of DH. Make DE = $\frac{1}{4}$ of DH, and FG = $\frac{1}{4}$ of FJ. Join G and E (not shown), bisect and produce to meet 1-3 at 1_0. 1_0 is equidistant from E and G, and is on the line of centres we desire to use for the *second* curve, which must be "similar to" but "smaller than" the first.

Project the dotted square parallel to 1, 2, 3, 4 from 1_0 and two other dotted squares the same distance within their counterparts in the first series. (Strictly, an objection might be raised to the latter procedure, but it gives so good a result (approximation) as to be quite acceptable.) You are now in

a position to complete the volute by repeating the procedure. Do this, following the inner curve in detail from the diagrams.

Small and decorative scrolls are often set out free-hand, and may advantageously be so produced by an artist; but the volute and scroll for architectural work of large size usually requires the mechanical accuracy possible only by methods such as we have described.

The following examples will provide the necessary and careful practice required at this stage.

EXAMPLES ON SCROLLS AND VOLUTES

(1) Draw a single line scroll similar to Fig. 258 when A8 = 16 ins., and the scroll projects to the *right* of the handrail. Place a curve parallel to this at a distance of $3\frac{1}{2}$ ins. within it, and continue it until the two meet. Your plan is then suitable for a handrail scroll.

(2) Set out a similar scroll to Fig. 258, when the main handrail line is in a horizontal position on the paper, and the diameter = 18 ins. Then divide the diameter into 10 parts instead of 8; construct the volute on the same principles, noting the difference in the rate of approach to the centre. Complete a handrail scroll for a handrail 4 ins. wide.

(3) Lay out the scroll step and handrail of Fig. 259 for a step 12 ins., instead of the 10 ins., as there shown. Place the plan with the scroll at the top of your paper, and the handrail leading vertically from it. Let W of rail be 4 ins., and the rest proportioned as before.

(4) Fig. 260 lends itself to the setting out of any wreathed scroll, whatever the width of step, because C_0 is the "centre" of the step, and $C_0 1$ is $\frac{1}{8}$ the width of the step. Assume a step 11 ins. wide, and a handrail $3\frac{1}{2}$ ins. wide; reverse the position of the plan, and lay out the scroll outline to the same condition and proportion of centres.

(5) An ionic volute is 14 ins. diameter. To any suitable scale determine the outline of the volute, whose eye is $\frac{1}{8}$ the diameter, fillet at broadest part to be $\frac{1}{3}$ of the expansion of the third convolution. DH in Fig. 261 is the expansion referred to. Make the setting-out eye coincide with the eye of the finished volute. Adopt as large a scale as your compasses will allow. It must be at least half size.

CHAPTER XXII

TRACERY

In a previous chapter we have referred to "tracery." This work, like the volute, is a special application of the tangential connection of circles, but there is no uniformity about the result.

Tracery may be defined as a series of curved and straight mouldings, generally symmetrical in design and arrangement, grouped in such a manner as to present a foliated (leaf-like) appearance, the latter being secondary in effect to the main outlines of the mouldings, which are usually assembled on the basis of some geometrical design.

Tracery may be applied to stone and wooden windows and doorways, screens, panelling, etc., and in its best form consists of a series of main members following some design with curved projections enclosing recesses of leaf-like form.

The curved recess is called a "foil," and the projection dividing two foils a "cusp" (*see* Fig. 279). The outline of a foil is not always an arc of a circle, but may take any form dependent on the exigencies of the design. Fig. 277 shows how "foils" vary in shape, and also the "cusps" dividing them.

Before entering on the setting out of "detail" in tracery, we must make sure that we understand the problems arising in its execution. We have before drawn attention to the "locus," and illustrated its value in solving fundamental problems. The application will be evident as we proceed, and we only need to draw attention to "shorter and better" methods *for use in special circumstances.*

Fig. 263 illustrates the direct method of constructing a circle to touch another circle and a given line tangentially, the latter at a "given point."

The circle centre A is given and the line on which it rests. A second circle, centre C, is required to touch the line at P, and also the given circle where it may. Drop a perpendicular to the line from P and a parallel line from A

to B. Join BP cutting the given circle at D. Join AD and produce to cut the perpendicular from P at C. C is the required centre, and (CP or CD) the radius, for ABD and CPD are similar triangles. AD = AB, therefore DC = PC. The latter are both normals, and AC is the "common normal" to the two circles because it joins their centres. (We have previously solved this type of problem by a "locus." Compare and draw your own conclusions.)

If the given circle be drawn away from the line another phase of the problem appears, for a circle may then be drawn to touch at P and "enclose" the given circle (*see* Fig. 263*a*).

The principle adopted is the same as we previously employed in constructing compound curves, *e.g.* the oval sewer section.

A perpendicular is dropped to the line from the centre O of the given circle, and a parallel drawn through P which must contain the required centre.

Set off the radius *r* from P at M; join MO and bisect it, letting the perpendicular bisector cut PM produced at C. Join CO and produce to Q. CQ is the radius and the common normal at Q, therefore the point of contact.

The use of this type of problem is seen at Fig. 264 on the right, where a simple tracery outline is here composed of an enclosing semi-circular arch with two secondary ribs AC and CB, also semi-circular, and on the same springing.

A circular filling is required to the spandrel portion (APBC) to touch the two semi-circles and the crown of the main arch. Now because the enclosed circle here must be smaller than the enclosing one the point P might be conceived as equivalent to lying on a straight line at the crown. Then we know that the required centre lies on the vertical through P, because this divides the elevation into symmetrical parts, hence, complete the circle CBM and drop the line EM parallel to PC.

Join PM cutting the circle at N, which is the point of contact. Join EN, and produce to intersect the centre line at F, which is the required centre. FP (or FN) is the required radius. The problem may be conceived as an application of Fig. 263 or of Fig. 266. It embodies both conditions.

On the left of Fig. 264 we have the first application of the locus to tracery work. It is fairly evident that the required circle must have its centre equidistant from the two curves APB and AC, hence a series of points has been determined, each equidistant from the two circles. A convenient method of setting out for the determination of these points is shown. Produce the springing to G. Set off AG = the radius of the main arch, AC. Mark off any uniform divisions as AL, LK, KJ, JH between A and G.

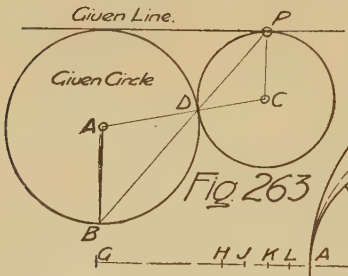


Fig. 263

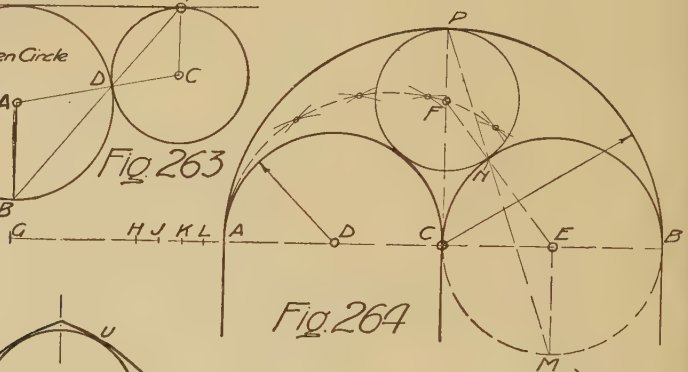


Fig. 264

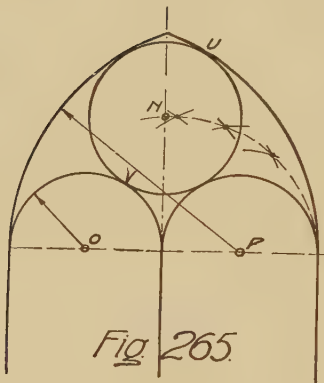


Fig. 265

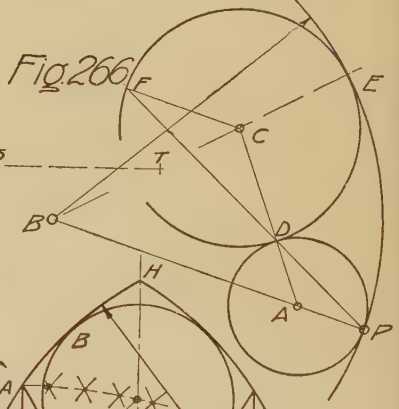


Fig. 266

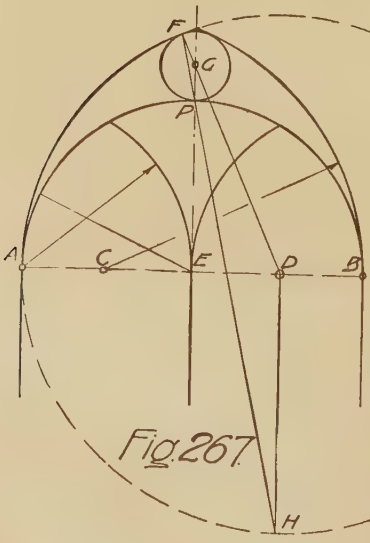


Fig. 267

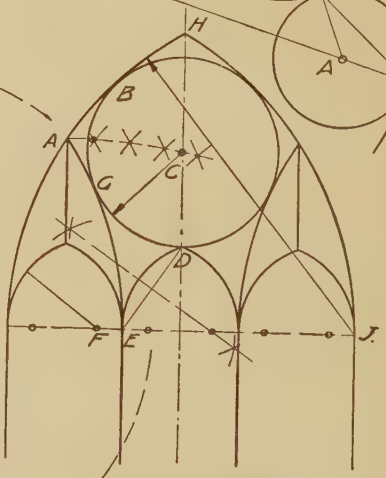


Fig. 268

Taking D as centre, strike a series of arcs with radii successively equal to DL, DK, DJ, DH. Then with centre C and radii successively equal to the differences between the "previous radii" and DG, strike another set of arcs to intersect the first, thus determining the points on the locus. Trace the fair curve to intersect the centre line at F which result agrees exactly with that of the "direct method."

Fig. 265 is a somewhat similar problem to Fig. 264, except that the enclosing arch is a pointed one. The enclosed circle forming part of the design touches four curves instead of three, but, being symmetrical, the problem requires a circle to touch the two given circles (on one side of the centre). We do not know "either" point of contact, nor the radius. Hence a locus is necessary. As this agrees with Fig. 264 in purpose, method of setting out, and result, it needs no special description.

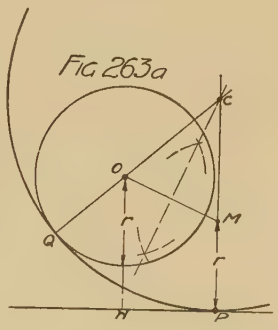


Fig. 266 shows a problem of the same "type" as Fig. 263 with a variation of conditions. A circle is required to touch two given circles which are already in tangential contact at E. Given the circle, centre C, and the tangential arc EP, required a circle to touch the two, and the large arc at P.

Use similar triangles as before. Draw normal EC produced inwards, and also the normal at P. These meet at B, the centre of the large arc. Place CF parallel to PB. Join FP, cutting the given circle at D; also join CD and produce to A, then because ADP is similar to FCD, and FC, CD are equal, AD and AP are also equal, and A is the required centre.

Fig. 267 is another case where the main tracery lines are settled, and a circle, centre G, is to be enclosed in the spandrel portion at the top of the window. The reverse conditions of Fig. 263*a* are here evident. P is the one known point of contact; AF is part of the enclosing circle. The centre G and the second point of contact F are required.

Now this problem is of a type requiring much room to set out, and for this reason the circle is often settled by trial. If the required circle is small, it is correct to adopt the method of "trial and error," but if comparatively large the following procedure is conveniently applicable.

Complete the circle AF, centre D. Drop DH parallel to the centre line. Join HP, and produce to F. Join DF, cutting the centre line at G. G is the required centre, and

GF the radius, for FGP is similar to FDH. FD and DH were equal, hence FG and GP are equal.

In Fig. 268 we have the further application of a "locus." A design for the main tracery bars has been adopted in which a main enclosing arch is equilateral in outline, and the mullions of the three-light window are carried through to the main arch with the same curvature as the latter. The centres for these curves—*e.g.*, curve EGA—therefore lie on the springing line of the main arch, produced, and are not shown. The remaining pointed arches, forming heads to the outer "lights," are dependent for their rise on the placing of the centre circle. This latter must touch the enclosing arch and the "continued mullion rib" at points such as B and G. As the centre for this circle must lie on the axis of the window and touch at two unknown points B and G, bisect the circular angle HAE by determining points equidistant from each arc. Trace the "locus," being the fair curve AC. C is the required centre. To determine the "radius" and point of contact B, join JC and produce to B (not shown). BC is the radius. A similar construction for the point G is required if the exact point of contact is needed.

We are now in a position to study the detailed setting out of the foliations. Until the present we have been considering "main centre lines of tracery designs" on which the foliation is built.

If foliation is arranged in a circle with perfect symmetry and based on the triangle, square, or polygon, it is named according to the number of indents or foliations composing it; thus, the centre of Fig. 277 is called a "trefoil" (three foliations), Figs. 275 and 279 "quatrefoils" (four foliations), and Fig. 270 a "cinquefoil" (five foliations). A six-foiled circle is called a "hexafoil."

Some of the fundamental problems relating to the setting out of foliation in tracery were given in vol. i., pp. 73 to 77. These included the placing of semi-circles within the triangle, square, and hexagon, on the assumption that all the foils were semi-circles. This is, of course, an elementary condition, because the foils are more often larger than a semi-circle. Moreover, their practical detail (or differences in detail) cause many variations to be made in the size and proportions.

In Fig. 269 we have a further fundamental problem illustrated. It is desired to place a series of five circles within the large circle, each touching two others and the enclosing circle tangentially. (This forms the basis of many foliation problems.)

Divide the circle into twice the number of sectors as compared

with the number of circles to be enclosed. We have "five" circles, therefore "ten" sectors. (You may do the division by trial, or use your elementary method of constructing a pentagon within the circle whose diagonals will serve your purpose.)

Now decide on the position which one circle is to occupy with its centre (as C, Fig. 269) on one of the diameters. Draw a tangent AB at the end of this diameter, and produce the radii bounding the sectors on each side to meet the tangent at A and B. Triangle ADB is the containing triangle for *one* circle, for, if the circle touches the straight line AB at E, it is also tangential to the circle at the same point. Determine C equidistant from each side of the triangle by bisecting the angle A. Drop CF at 90° to AD. C is the centre and CF the radius required. Transfer DC—by the dotted circle—to all the diameters, thus placing the remaining centres. The faint circle through F is the circle along which the contacts of all the enclosed circles occur, and if the arcs within this be ignored or removed a "cinquefoil" in single lines remains.

Fig. 270 illustrates the complete detail of the finished work taking into account the setting out of the foils, the allowances for fillets or margins on the faces of the bars, the satisfactory finish to the ends of the cusps, and the removal of spandrel surfaces between main lines and cusps by triangular sinkings, called "eyes."

Observe that we have to keep in view the fundamental idea of the cinquefoil, and yet give width to the main circular bar and to the cusps.

Try to follow this description of the order of procedure :—

(*a*) Draw the enclosing circle and the flat F of the main circular bar. (*See* detail at side of Fig. 277 for shape of this bar.)

(*b*) Divide the circle into 10 sectors. Notice the lines on which "centres" lie, and on which "cusps" will be formed.

(*c*) Decide the width of the cusp at the point. It may be anything from $1\frac{1}{4}$ ins. down to $\frac{1}{2}$ in.; assume $\frac{7}{8}$ in. On each side of the diameter containing the cusp measure half this width and place dotted lines as shown below D and between D and B. At M measure the width of the hollow mould on the edge of the bar—as detail, Fig. 277.

(*d*) Draw AB through the measured point P at 90° to PD to meet the sides of the triangle enclosing the inside curve of the foil. Then BDA is the correct enclosing triangle for the foil.

(*e*) Bisect angle A and produce to cut EP at C. C is the centre for the arc of the foil.

(*f*) Carry the distance DC round the whole figure, and draw the inner arcs stopping them on the second dotted circle, at the points of contact. Form an angular point at the cusp, enclosing, at your choice, 90° to 110°. Arrange this on one cusp, and carry its position to the rest of the diagonals by the faint dotted circle.

(*g*) Carry the edge of the "flat margin" round each foil to meet its neighbour on a diameter, and trace the "mitres" at the ends of the cusps. Include the flat margin by parallel curves while maintaining the width all round the main bar.

These flats leave a curved triangular spandrel which is too small to perforate and too large to leave unrelieved. To give the appearance of being part of a bar, a deep V groove is cut as a chamfer from each angle forming a long mitre, and the three V's meeting at the centre complete a triangular sinking of the form you have seen in "chip carving." It is called an "eye." The shaded eye to the right of the figure and the section below Fig. 279 should make the detail clear.

The cusps of Fig. 270 are emphasised and clear, but this is not always desired. Refer, for instance, to Fig. 271, which is arranged as a detail of the centre of Fig. 268, including the junctions at D and E, and incidentally illustrating a series of flat cusps. The flatness refers to the angle between the two intersecting arcs "forming the cusp," and not to any mouldings connected with it.

In this detail, centre lines of the main members are drawn dotted, and the main outlines of the mouldings are placed equally on each side.

The inner arch is required to be foliated by introducing three circular cusps on each side, and, because the head is pointed, the top foil will differ from the rest. Although not so treated in this example the "head foil" may be emphasised by making it larger than the rest. Our illustration is really smaller, and you must carefully consider this point as a fault often arising.

Having decided the number of cusps to be inserted (say three), the procedure is as follows:—

From the crown of the pointed arch choose a point M "near" the crown or "distant" from it according to the emphasis desired for the top foil. Join M to C. Divide the

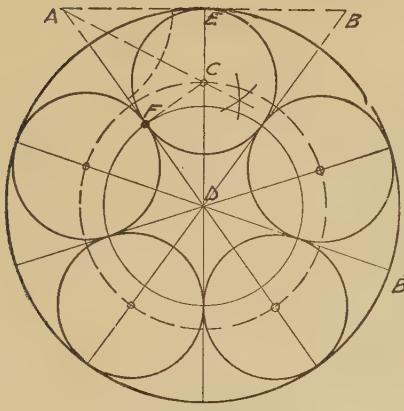


Fig. 269.

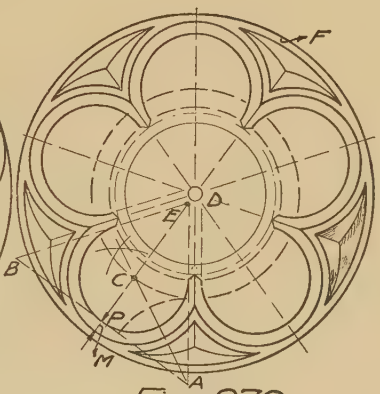


Fig. 270

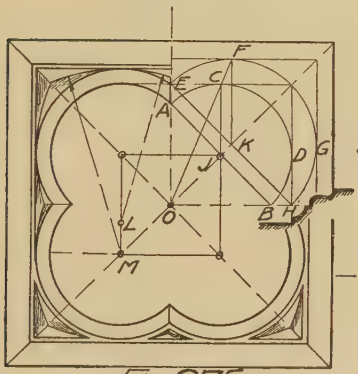


Fig 275

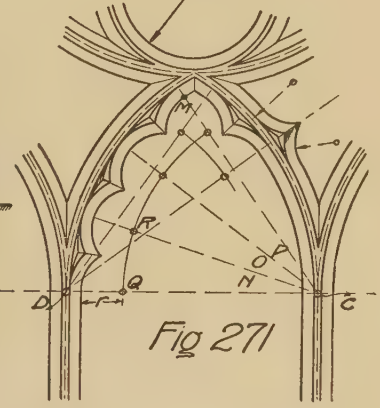


Fig 271

Fig 272.

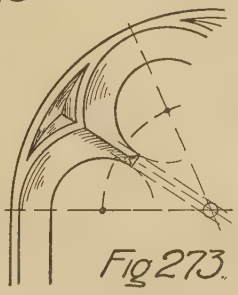


Fig 273.

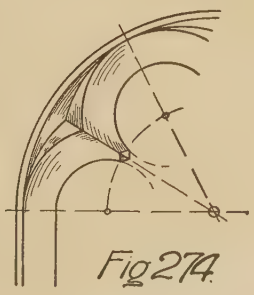


Fig 274.

angle MCD into three equal parts. The centres for the foil curves lie on these lines. Now make a trial for the cusp outlines. Working from the springing lines and from the outside edge of the mullion and bar, choose a radius *r*, and with Q as centre strike the first cusp curve.

From R strike a similar curve downwards to cut the first. According to your choice of finish at the point of the cusp make trials until you have located Q to suit your purpose. Transfer the chosen centre to the remaining radii, and complete the inner curves; the rest are parallel.

If the angle at the point of a cusp is required to be "sharp," the enclosed angle between the tangents to the curves at the point should be at least 90°. In example of Fig. 271 the angle referred to is practically 90°.

Examine the opening above the mullion to the right. A cusp has been arbitrarily inserted here to illustrate the treatment of the spandrel within such as EGD of Fig. 268. One such cusp would be placed on each side of the spandrel opening, giving an appearance which may be generally gathered from completed work in Fig. 277.

Now the cusp last referred to in Fig. 271 is "sharp at the point," and although its appearance is better than those of the central opening, if constructed in stone it would soon be "weathered" away.

In order to illustrate further differences in cusp details three figures have been inserted, viz., Figs. 272, 273, and 274, which give distinctly typical cusps.

The first shows the best architectural form in present use, suitable for working on a chamfered bar. The point is "prominent" but rounded, and the eye is formed at a distance from the face of the bar by making the main chamfer start at the site edge of the flat top, and the cusp to spring away from it at a lower level. On the opposite edge of the eye a sharp angle occurs, and the bevel is constant all round, being the same as the chamfer.

The advantage of this form is that the cusp in no way detracts from the main outline of the tracery, but serves to decorate its own opening rather than add to the ornateness of the main face of the tracery bars.

Fig. 273 has the margin of the main bars carried round the foils and the eye cut within. The eye becomes smaller and less emphatic, and the "face outline" appears too much foliated and involved.

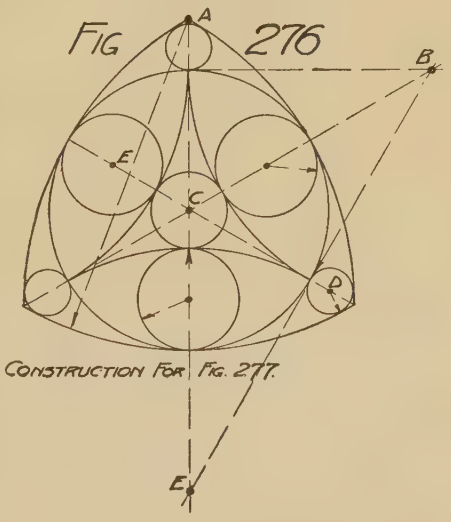
Fig. 274 is a common form with the eye sharp-edged within, but level with the main surface of the tracery bars. It is better than Fig. 274, but not so good as 272.

In the two latter cases the end of the cusp is finished by returning a small portion of the curve round the "square end," or a "pointed end," as required. This causes additional short mitres, but keeps the foils sharp in outline for the time being. Further merits and demerits of these details and their relation to complete architectural features must be left to future design classes.

Fig. 275 is an application of the principle explained in Fig. 100, vol. i., to the setting out of a quatrefoil on a wooden panel. The foils are semi-circles, placed in squares, the process of placing these being as follows:—

Determine the square that contains the "inside" curves of the foils. Draw its diagonals (dotted in the figure) and the semi-diameters OE and OH.

On EH describe a semi-circle, and enclose it in a square so that it touches at F and G. These points are accurately determined by the normal FK. Join FO, cutting the original square at C. Draw CJ parallel to FK. J is the required centre, and the semi-circle ACDB is the required one. Repeat the curve for the other quadrants, and complete the figure, designing your own moulding, but leaving the edge on which the cusping is worked, hollow, as shown in the section to the right.



CONSTRUCTION FOR FIG. 277.

The centres M and L have been used here for drawing the mitre lines for the "eyes" at the four corners and sides respectively. These are only approximations, but very satisfactory ones under average conditions. For the principle underlying these, *see* Figs. 278 and 279 and later description.

In Fig. 276 you will find the basis for the design of a tracery window, fully illustrated in Fig. 277. The design is based on the equilateral triangle and circle, and the dimensions required are shown on the latter figure and its details. The centres of the circles and arcs composing the design are ABCD and E, and these, suitably repeated, ought to be easily followed and used from the figure—276.

To apply this basic work to Fig. 277, which should be at least partially reproduced to a minimum scale of 1 in. = 1 ft., follow the appended procedure :—

1. Place all the centre lines in position as per Fig. 276. Let these be dotted or very fine lines.
2. Measure off the fillet or margin of the bars and the width of the moulding, and carry these parallel on both sides of the centre line round every bar (except the external line). Note that the main or enclosing curves should be set out as the *centre* of the "enclosing fillet"—not the *edge* of it.
3. Observe the type of cusp in each opening. (These are varied for illustrative purposes only.) Examine details in Figs. 278 and 279, and insert the same in the respective openings in the tracery. Notice particularly that the cusp points in the *irregular spandrel openings* "differ from any previous illustration." *Fill in all cusps of ONE type.*
4. Trace in the eyes of the cusps, observing that the point of intersection of the mitres should be the centre of the spandrel outline. Further, that the mitres of these cusps may be drawn by adopting a circular arc closely approximating to the locus bisecting the circular angle. Thus, in Fig. 278 place A midway between B and C. Join AD, bisect it, and produce to meet the radius from D at E, which is the required centre for the approximate arc.

With great care fill in the detail, noting that many intersections of curved mouldings occur, having various radii, and that these are not straight, as proved in the chapter on intersection of mouldings (chap. xvii.).

You will probably find that mitres at "curved bars" and "eyes" will suffice freehand.

Figs. 280 and 281 illustrate a small wheel window, Fig. 280 showing how to determine the centres for the connecting curve in the main outline, and the two together supplying sufficient information for its completion.

Set out to the details supplied, taking the main diameter 10 ft. at the outer edge of the fillet, and the inner circle 5 ft. to 5 ft. 6 ins. diameter outside.

The basis of the design is a combination of the equilateral triangle and circle, and only one serious difficulty occurs, viz., the placing of the tangential arc LK, shown in full detail at Fig. 280. Let circles whose centres are H and J be drawn ;

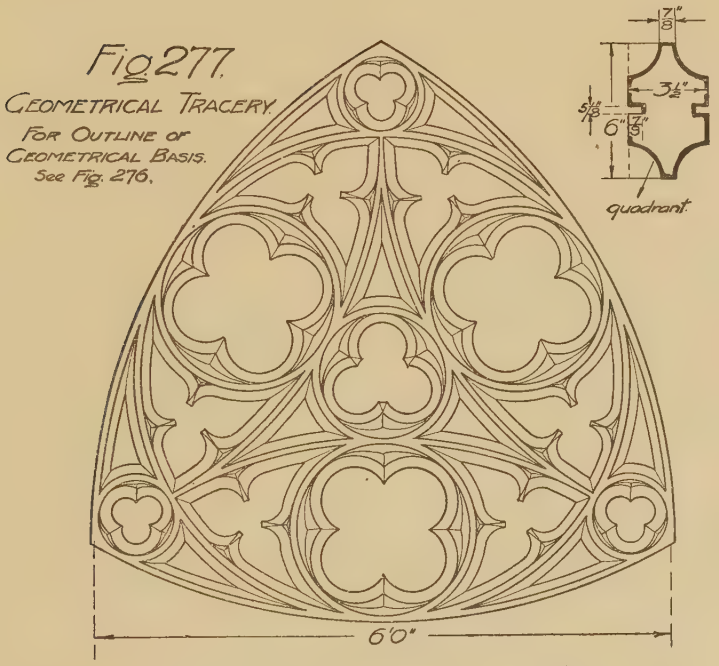


Fig. 277.
GEOMETRICAL TRACERY.
FOR OUTLINE OF GEOMETRICAL BASIS. See Fig. 276.

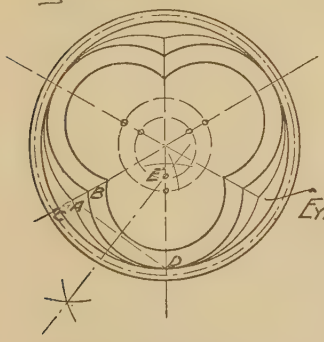


Fig. 278.
DETAIL OF TREFOIL.

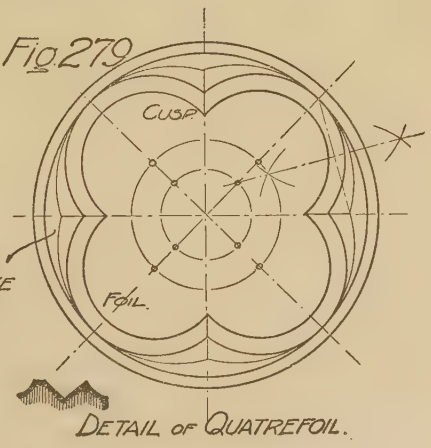
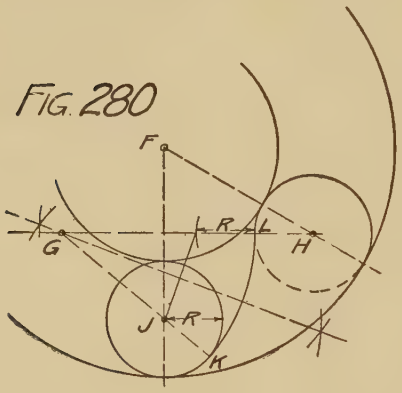
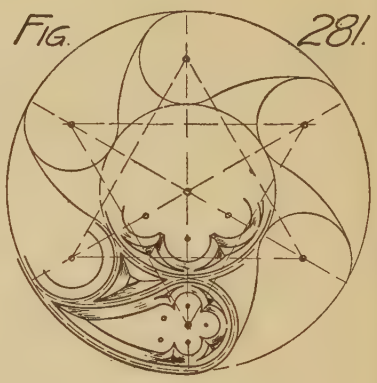


Fig. 279.
DETAIL OF QUATREFOIL.

the sector enclosed between the radii FH and FJ is $\frac{1}{6}$ of the circle. If curve LK joins tangentially at L its centre must lie on the radius HL produced, and be equidistant from both curves; therefore, set off R from L towards G, join RJ and





bisect it, producing to cut HL produced at G. G is the required centre.

Draw the common normal GK through J, place the connecting curve, then arrange the detail of bars and cusps, as before, to agree with Fig. 281.

Exercise Work in Tracery

As exercises in tracery the student should first work out in detail all the examples given in this chapter (not necessarily repeating similar parts), then add to his knowledge by measuring and sketching tracery windows within his reach, and draw out complete details of the whole or suitable parts.

While trying to apply the foregoing principles he will find many variations of our problems, and also new difficulties, but if the work in this chapter has been done conscientiously, very few insurmountable difficulties should be met even in complicated work.

You have learnt nothing in the way of "construction" of tracery, and you are advised not to attempt it at the present stage. The sole concern here is with the geometrical problems arising in the setting out of tracery.

CONCLUSION

BECAUSE of the variety of trades and professions, representatives of whom are gathered together for instruction in "Building Classes," it is difficult to treat the geometry of building so that every student gets his proper share of the work which is most important to himself.

Hence the foregoing chapters cannot be said to be complete, because this work may only include what space allows, and what may be generally taught with interest to many grades of students.

The sections chosen have been treated in detail sufficient for *most* students, even of the more brilliant type. Nevertheless, keen students are advised to prosecute further, where necessary, those branches of study which are apparently of most importance to them.

In any case, do not let your study develop into "casual reading," which in the study of geometry is "worthless." A student must be able to place his knowledge and ideas on paper, and, as far as possible, without assistance, after studying the underlying principles of the subject in hand.

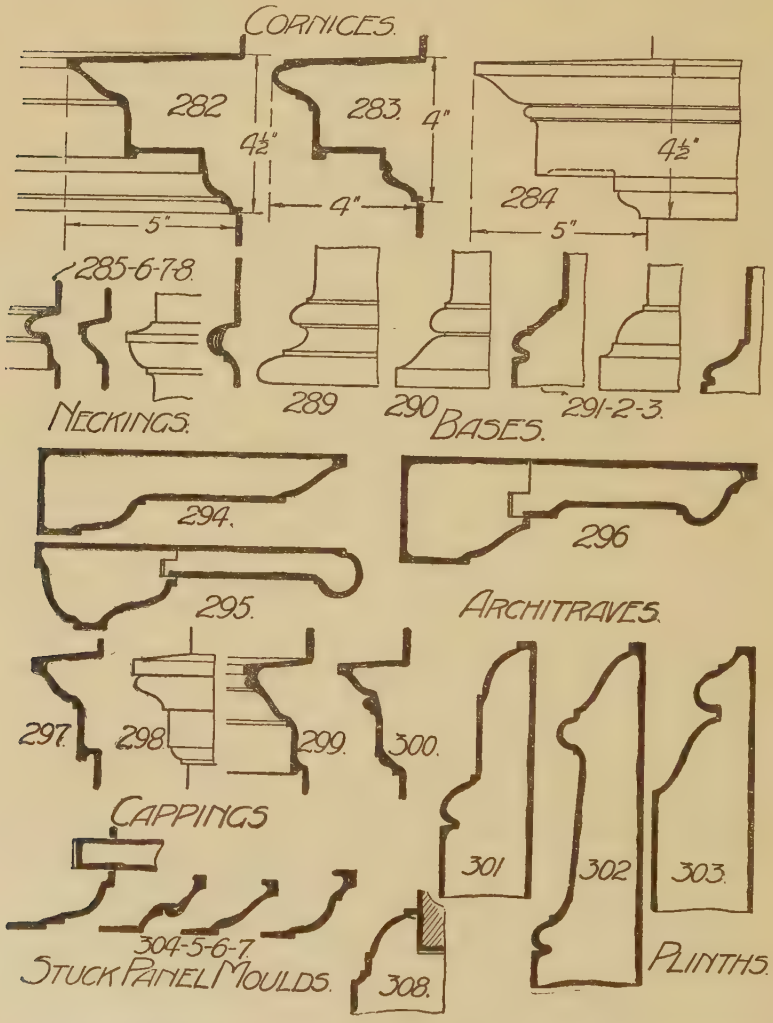
The only satisfactory mode of procedure is to attempt every exercise that time will allow, and as the test examples in this volume, appended to each chapter, are directly based on the explanatory text of foregoing chapters, the time will be profitably spent as a test of successful study.

Some usual parts of the subject of geometry have been intentionally omitted. These include: "Graphic arithmetic, vectors, pictorial drawing, and isometric projection," the intention being that these are better taught in connection with "construction" and "mechanics."

Trigonometry has also been treated in a very elementary form, because of lack of space and its limited value to the artisan student.

For the assistance of students in the choice of designs for mouldings required in the geometrical processes involved in enlarging mouldings, designing raking moulds, etc., a

page of sections has been added herewith, designed for special purposes and referenced.



A useful exercise may be found in naming their constituent parts and designing other mouldings suitable for planting on door panels, for picture mouldings, dadoes, built-up plinths, etc., to harmonise with some choice of type from the illustrations.

MATHEMATICAL TABLES

[*A copy of these Tables will be supplied to each candidate at the Examinations in Practical, Plane, and Solid Geometry, Machine Construction and Drawing (Higher Stage), Building Construction (Higher Stage), Naval Architecture, Practical Mathematics, Applied Mechanics, and Heat Engines.*]

USEFUL CONSTANTS

1 Inch = 25.40 millimetres.

1 Gallon = .1604 cubic foot = 10 lbs. of water at 62° F.

1 Knot = 6080 feet per hour = 1 nautical mile per hour.

Weight of 1 lb. in London = 445,000 dynes.

One pound avoirdupois = 7000 grains = 453.6 grammes.

1 Cubic foot of water weighs 62.3 lbs.

1 Cubic foot of air at 0° C. and 1 atmosphere, weighs .0807 lb.

1 Cubic foot of hydrogen at 0° C. and 1 atmosphere, weighs .00559 lb.

1 Foot-pound = 1.3562×10^7 ergs.

1 Horse-power-hour = 33000×60 foot-pounds.

1 Electrical unit = 1000 watt-hours.

Joule's equivalent to suit Regnault's H, is $\begin{cases} 774 \text{ ft.-lbs.} = 1 \text{ Fah unit.} \\ 1393 \text{ ft.-lbs.} = 1 \text{ Cent. ,,} \end{cases}$

1 Horse-power = 33000 foot-pounds per minute = 746 watts.

Volts × ampères = watts.

1 Atmosphere = 14.7 lbs. per square inch = 2116 lbs. per square foot = 760 mm. of mercury = 10^6 dynes per square centimetre nearly.

A column of water 2.3 feet high corresponds to a pressure of 1 lb. per square inch.

Absolute temperature, $t = \theta°$ C. + 273° or $\theta°$ F. + 460°.

Regnault's H = 606.5 + .305 $\theta°$ C. = 1082 + .305 $\theta°$ F.

$pu^{1.0648} = 479$.

$\log_{10} p = 6.1007 - \frac{B}{t} - \frac{C}{t^2}$,

where $\log_{10} B = 3.1812$, $\log_{10} C. = 5.0882$.

p is in pounds per square inch, t is absolute temperature Centigrade.

u is the volume in cubic feet per pound of steam.

$\pi = 3.1416$.

One radian = 57.30 degrees.

To convert common into Napierian logarithms, multiply by 2.3026.

The base of the Napierian logarithms is $e = 2.7183$.

The value of g at London = 32.182 feet per second per second.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	128	0170	0212	0253	0294	0334	0374	4	9	13	17	21	26	30	34	38
											4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	12	15	19	23	27	31	35
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89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
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93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
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95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
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·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
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·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Angle.		Chord.	Sine.	Tangent.	Co-Tangent.	Cosine.			
Degrees.	Radians.								
0	0	0	0	0	∞	1	1·414	1·5708	90
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5.6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6158	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45
			Cosine.	Co-Tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	

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